Posterior samples of source galaxies in strong gravitational lenses with score-based priors

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Mila

Linear Inverse Problems

An inverse problem consist of finding $\mathbf{x}$, a latent variable of interest, given a noisy observation $\mathbf{y}$.

$$\mathbf{y} = A\mathbf{x} + \mathbf{z}$$

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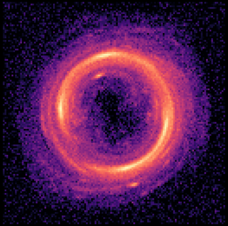
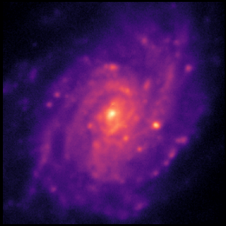


$= A$

$\mathbf{x}$



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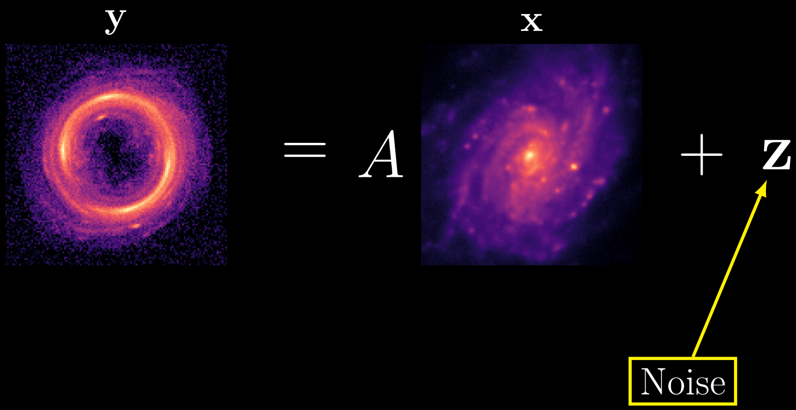

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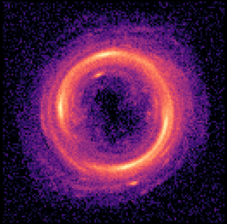

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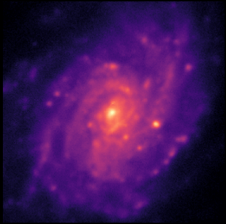


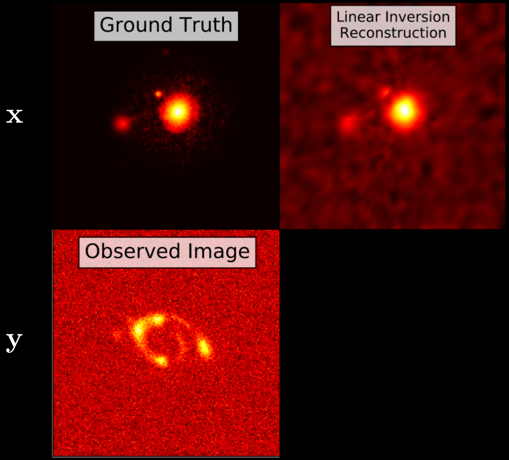
$+ \mathbf{z}$

Simulator

Noise

Solving the Linear Inverse Problem

Traditional method



Credit: Morningstar et al. (2019)

Bayes' theorem

The posterior is defined as

$$\overbrace{p(\mathbf{x} \mid \mathbf{y})}^{\text{posterior}} = \frac{\overbrace{p(\mathbf{y} \mid \mathbf{x})}^{\text{likelihood}} \overbrace{p_\theta(\mathbf{x})}^{\text{prior}}}{p(\mathbf{y})}$$

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Likelihood
Simulator ✓

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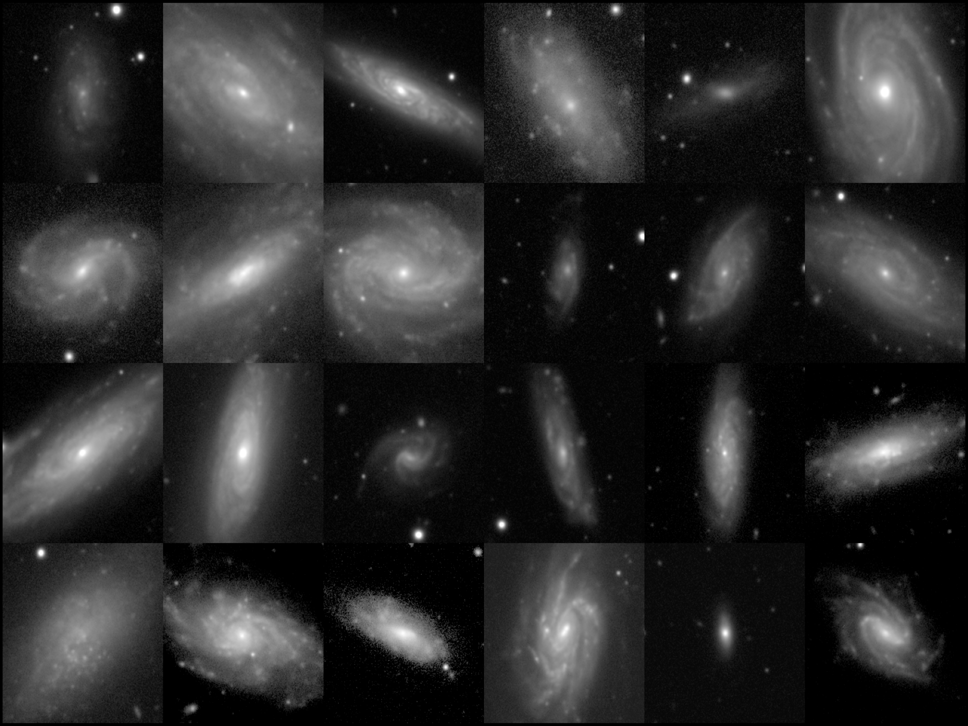
Likelihood
Simulator ✓
Noise statistics ✓

Posterior
Sampling ×

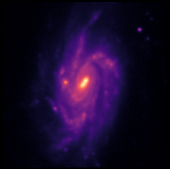
Prior
Closed form ×



Score-Based Modeling

Diffusion

Target distribution



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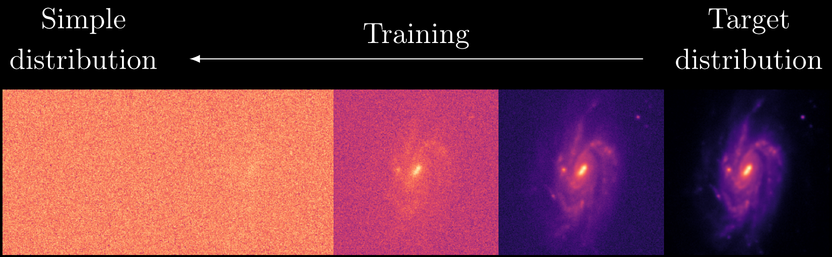


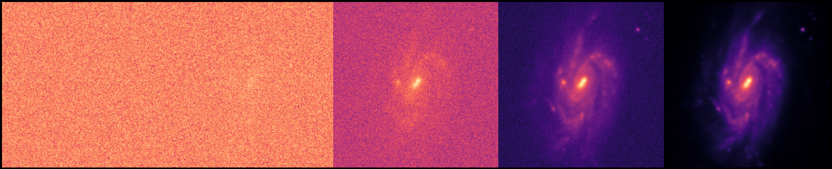
Score-Based Modeling

Diffusion

Simple distribution

Learn $\boldsymbol{\nabla}_{\mathbf{x}} \log p_\theta(\mathbf{x})$

Target distribution



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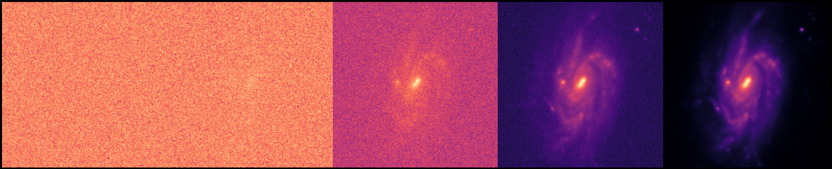

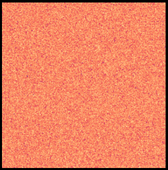

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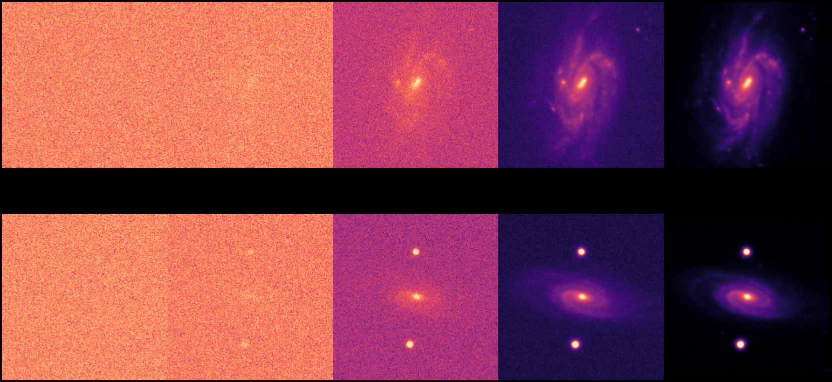


Sampling: follow the score

Score-Based Modeling

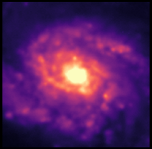
Posterior

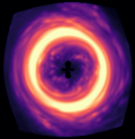
The score of the likelihood is all you need to sample from the posterior.

$$\underbrace{\nabla_{\mathbf{x}} \log p(\mathbf{x} \mid \mathbf{y})}_{\text{posterior}} = \underbrace{\nabla_{\mathbf{x}} \log p(\mathbf{y} \mid \mathbf{x})}_{\text{likelihood}} + \underbrace{\nabla_{\mathbf{x}} \log p_\theta(\mathbf{x})}_{\text{prior}}$$

Posterior sampling (iid)

Ground
Truth





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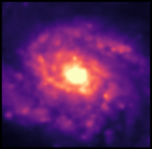

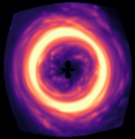

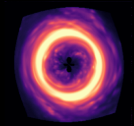

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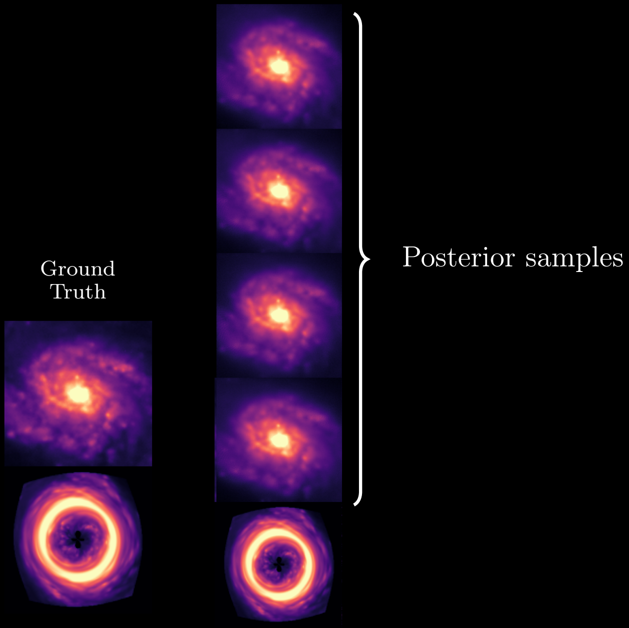

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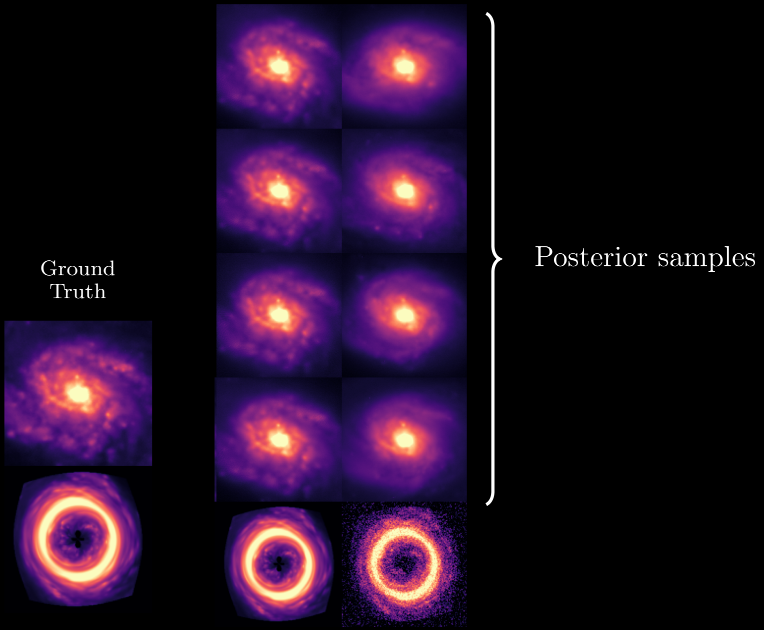
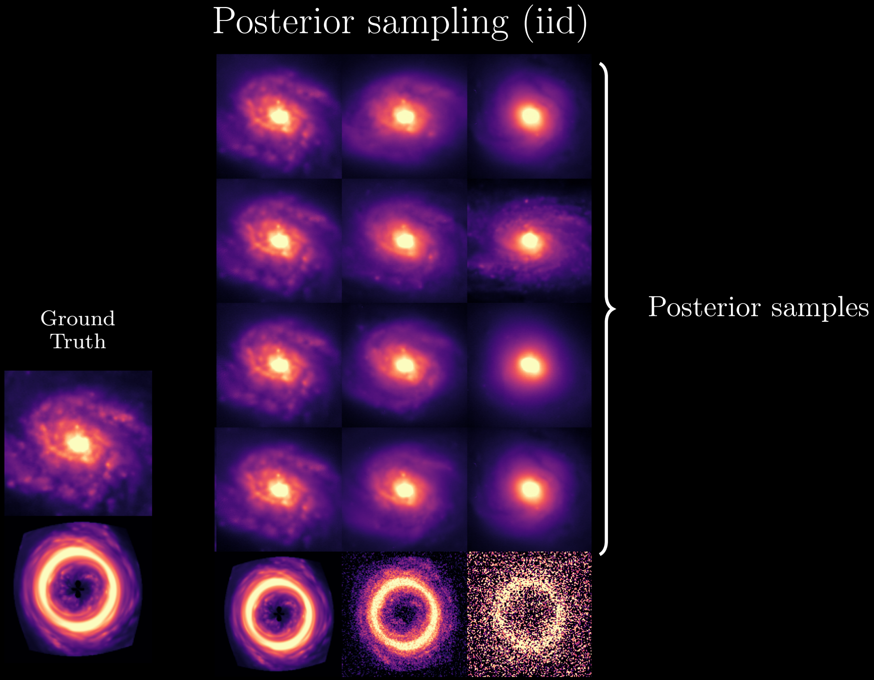
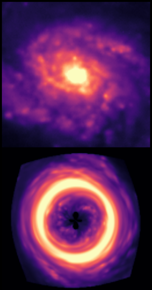


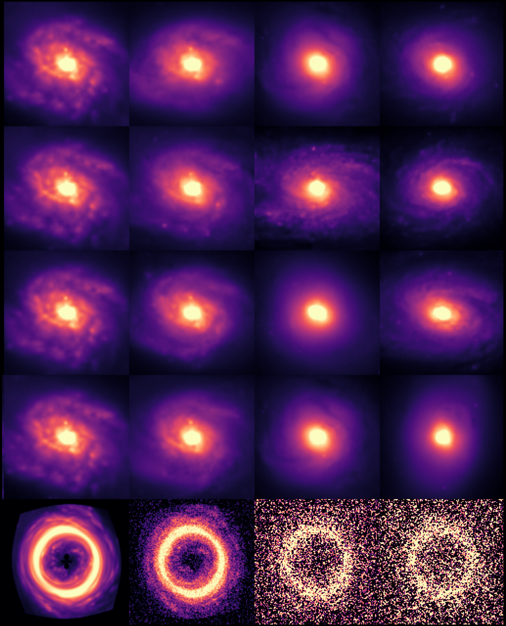


Posterior sampling (iid)
Ground
Truth
Posterior samples

Posterior sampling (iid)

Ground
Truth





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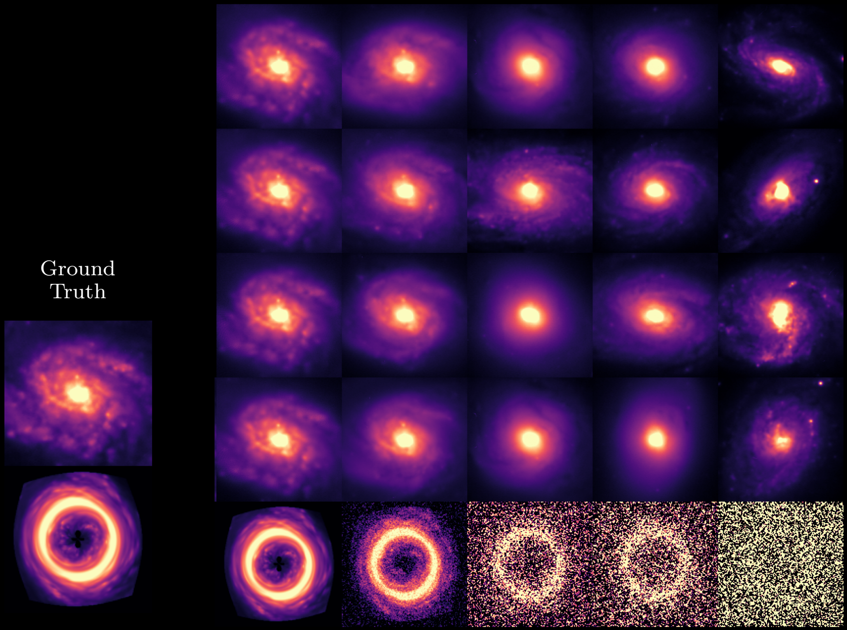
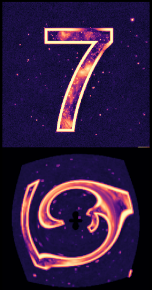


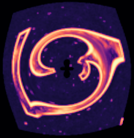
Posterior sampling (ood)

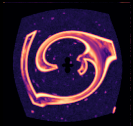
Ground
Truth



Posterior sampling (ood)

Ground Truth





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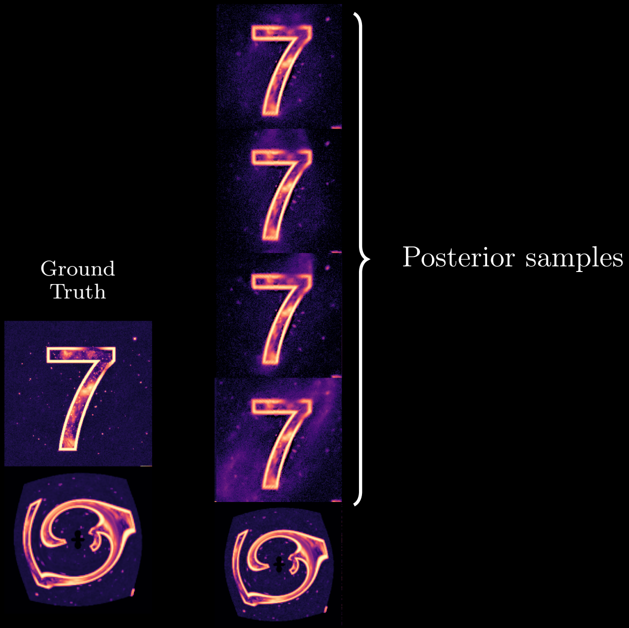

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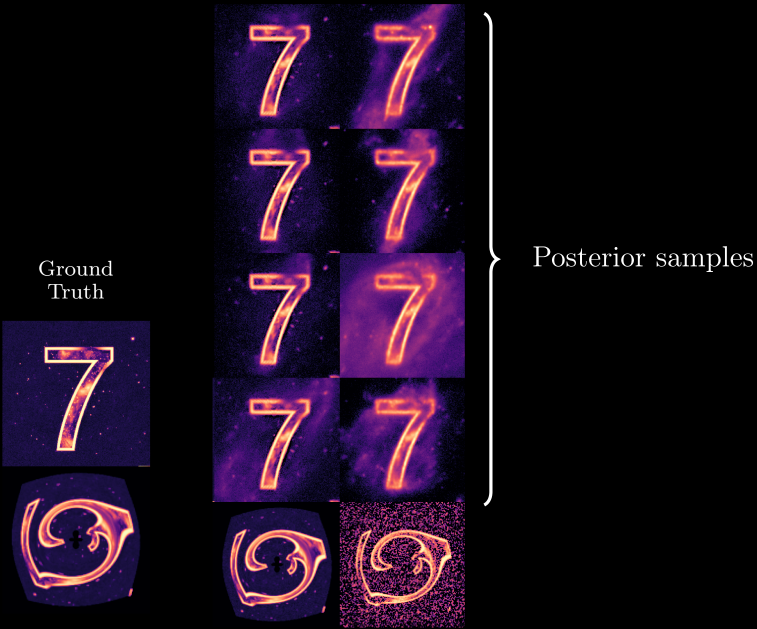
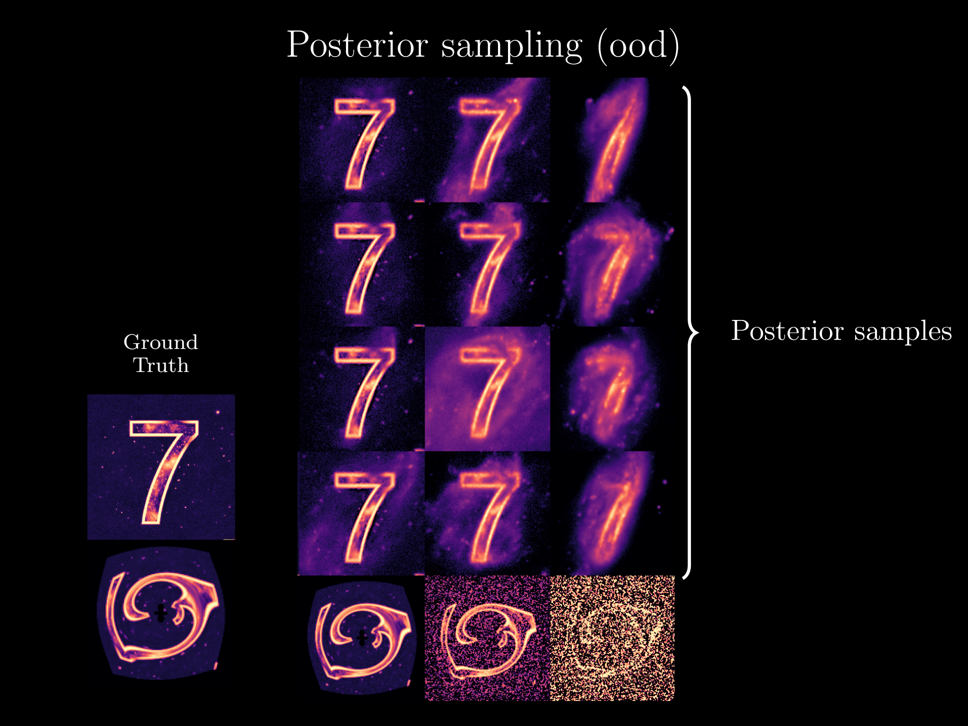
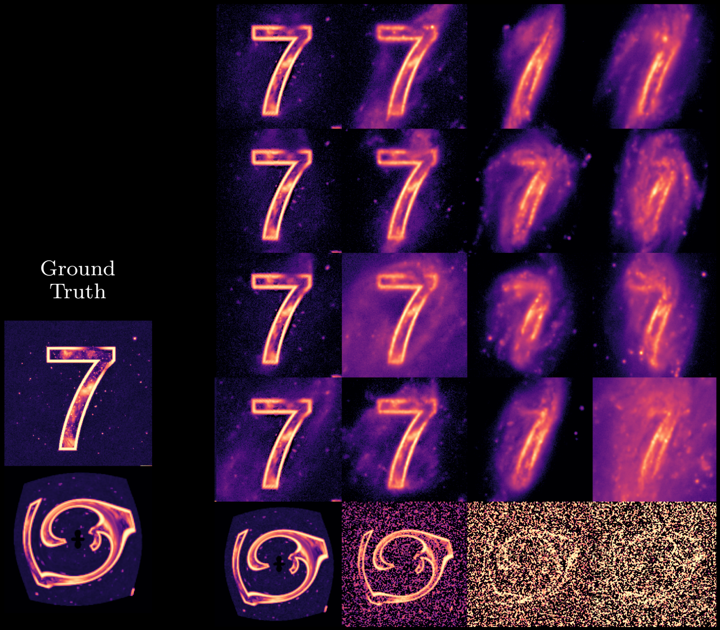




Posterior sampling (ood)
Ground
Truth
Posterior samples

Posterior sampling (ood)



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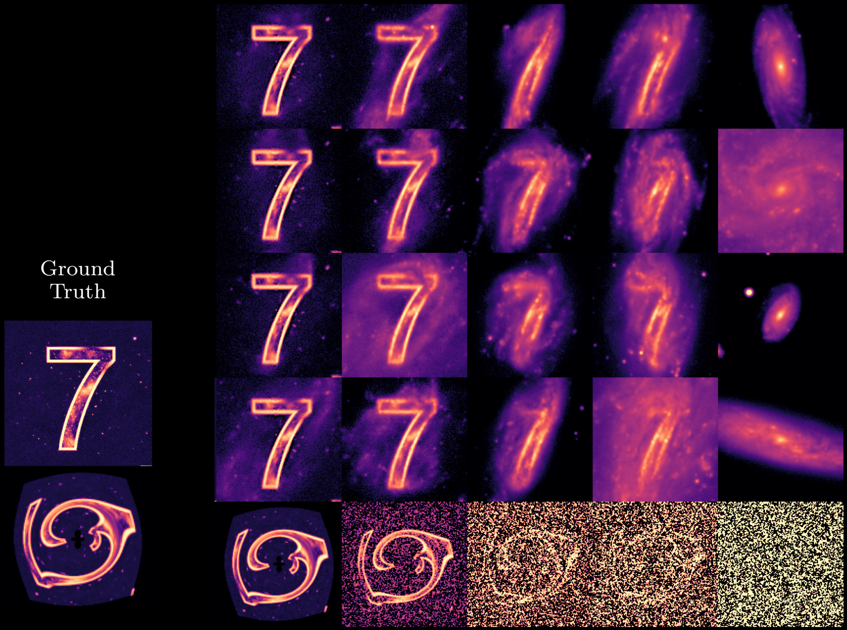


Learning the likelihood

Likelihood
Simulator ✓
$\boxed{\text{Noise statistics ✓}}$

Posterior
Sampling ✓

Prior
Learned from data ✓

Learning the likelihood

Likelihood

Simulator ✓

$\boxed{\text{Noise statistics ✓}}$

Posterior

Sampling ✓

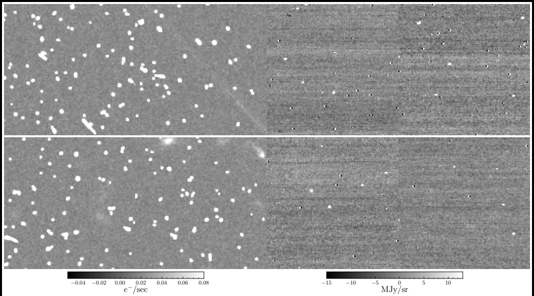
Prior

Learned from data ✓

HST *JWST*

Real

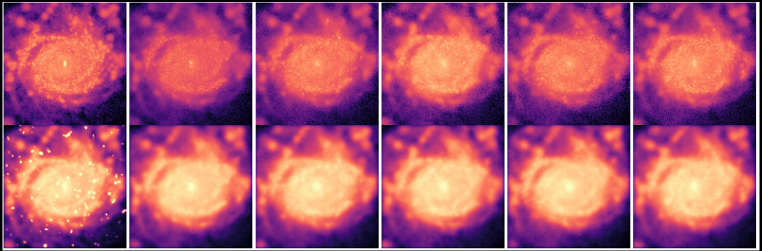
Generated



Learning the likelihood

Ground Truth

Posterior Samples



In Conclusion

We can do rigorous bayesian inference with highly accurate priors and likelihoods learned from data in high-dimensional spaces.

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Thank you

arXiv:2302.03046 arXiv:2211.03812