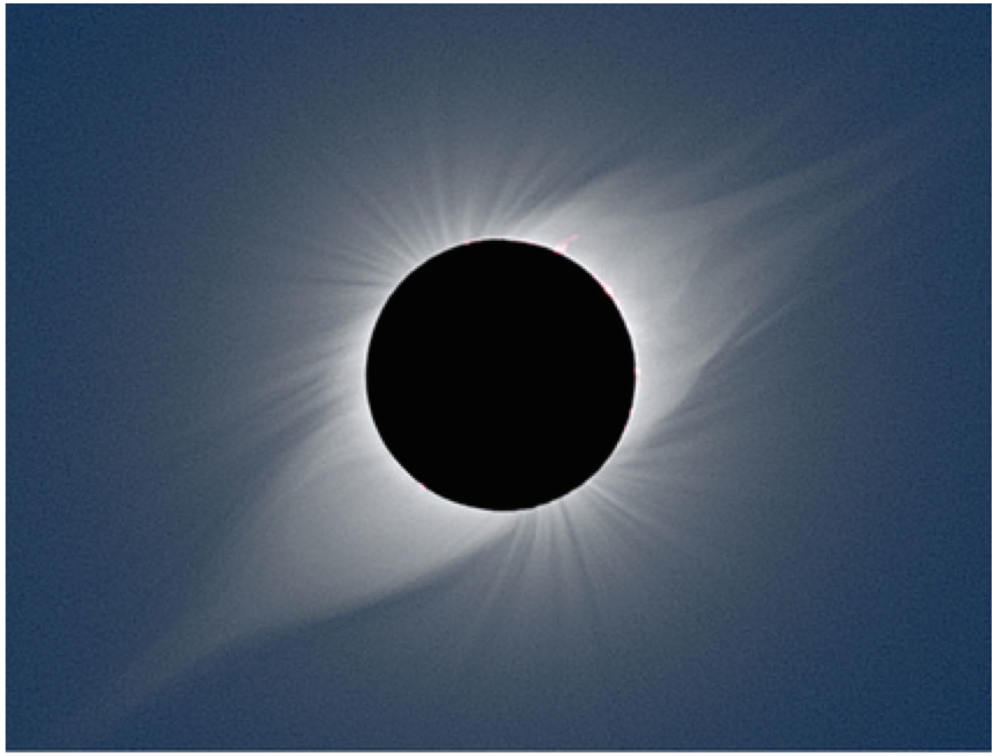

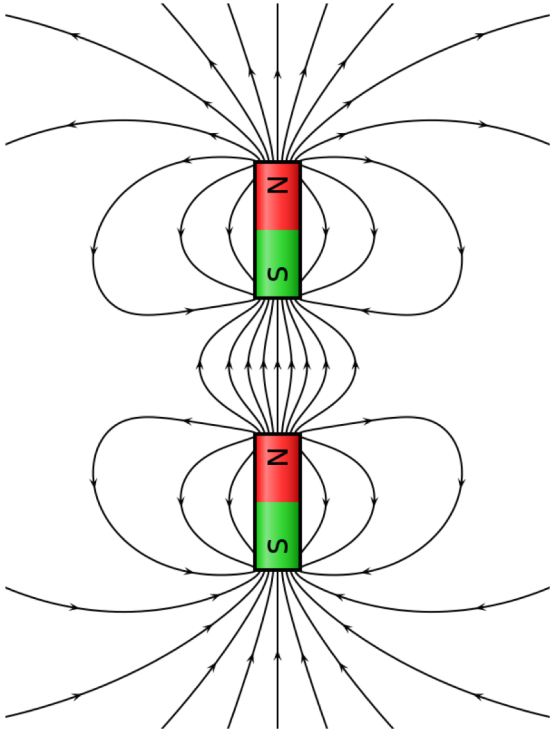

Two-Dipole Model of the Sun's Magnetic Field

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&

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Overview

Spatial Power Spectrum of the Photospheric Magnetic Field:

- Eliminating the vantage point effect
- Filling the polar data gaps
- Normalization of the spherical harmonic coefficients

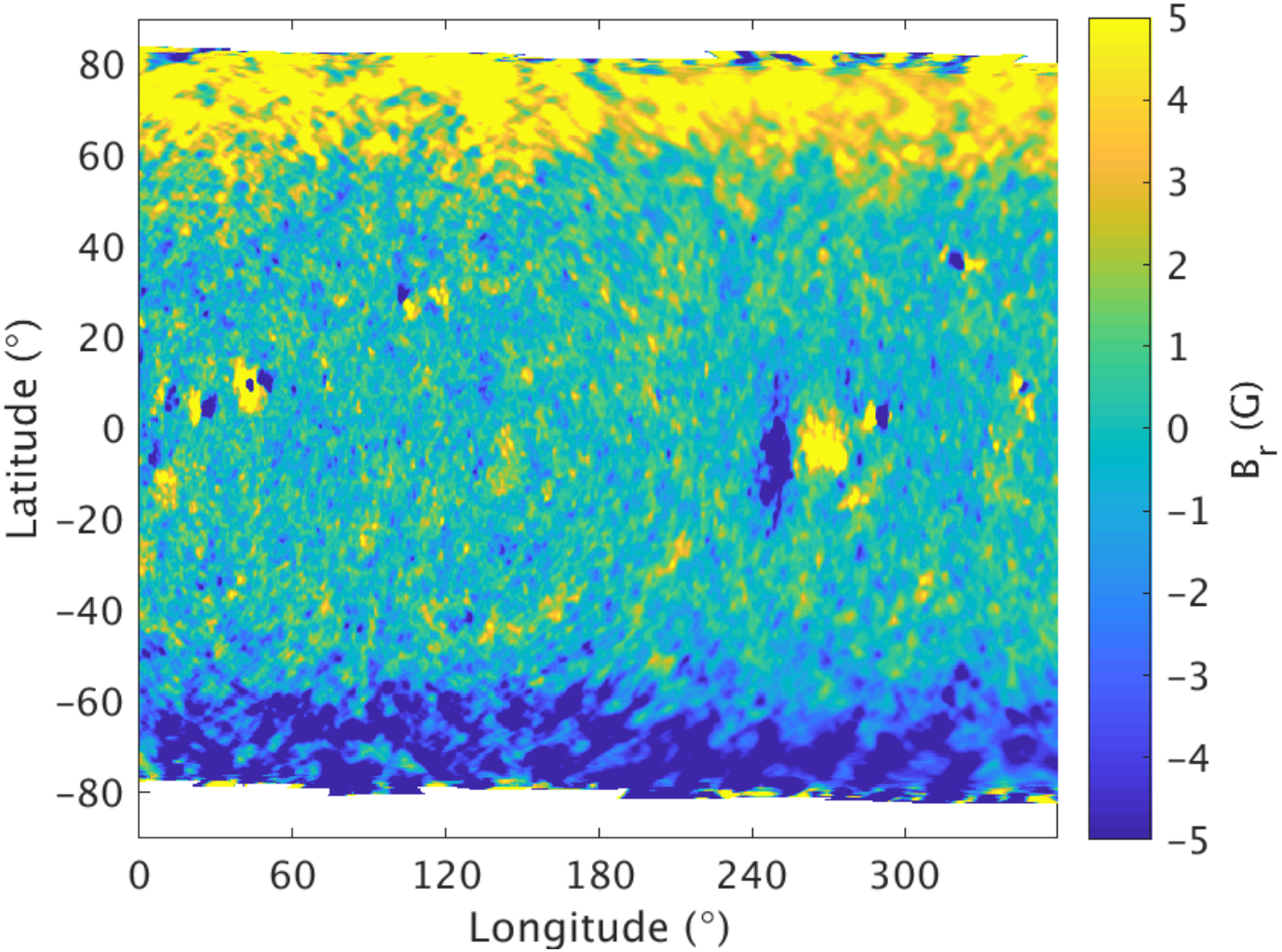
Two-dipole Model:

- Justifying the two-dipole approximation of the photospheric field
- Derivation of the potential of two eccentric axial dipoles
- Fitting the two-dipole model to observed harmonic coefficients

North-South Asymmetry:

- Is there a persistent north-south asymmetry during solar minima?
- What is the cause of the asymmetry?
- How does the photospheric asymmetry affect the coronal magnetic field and the interplanetary magnetic field at Earth?

Synoptic Map of the Photospheric Magnetic Field
MWO, CR 1910, June 1996



Radial magnetic field assumption in the photosphere:

$$B_r = B^{los} / \sin\theta$$

where B^{los} is the line-of-sight magnetic field component, and θ is the colatitude.

Definition of Harmonic Coefficients

Solar Physics:

$$B_{\mathrm{r}}(R_0, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} P_l^m(\cos\theta)(\mathrm{g}_l^m \cos m\phi + h_l^m \sin m\phi)$$

g_l^m and h_l^m are the harmonic coefficients of the spherical harmonics expansion of the radial magnetic field component in the photosphere.

Geomagnetism:

$$\Psi_{\mathrm{I}}(r, \theta, \phi) = R_0 \sum_{l=0}^{\infty} \left(\frac{R_0}{r}\right)^{l+1} \sum_{m=0}^{l} P_l^m(\cos\theta)(\mathrm{g}'^m_l \cos m\phi + h'^m_l \sin m\phi)$$

g'^m_l and h'^m_l are the harmonic coefficients of the spherical harmonics expansion of the internal potential, also known as **Gauss coefficients**.

Neglecting external sources in the photosphere, the two coefficients are related as:

$$\mathrm{g}_l^m = (l+1)\mathrm{g}'^m_l \qquad\qquad h_l^m = (l+1)h'^m_l$$

Calculating the Zonal Gauss Coefficients From Latitudinal Profiles

For photospheric magnetic field data with a longitude-latitude grid of $N_\phi \times N_\theta$, the zonal ($m = 0$) Gauss coefficients can be expressed as

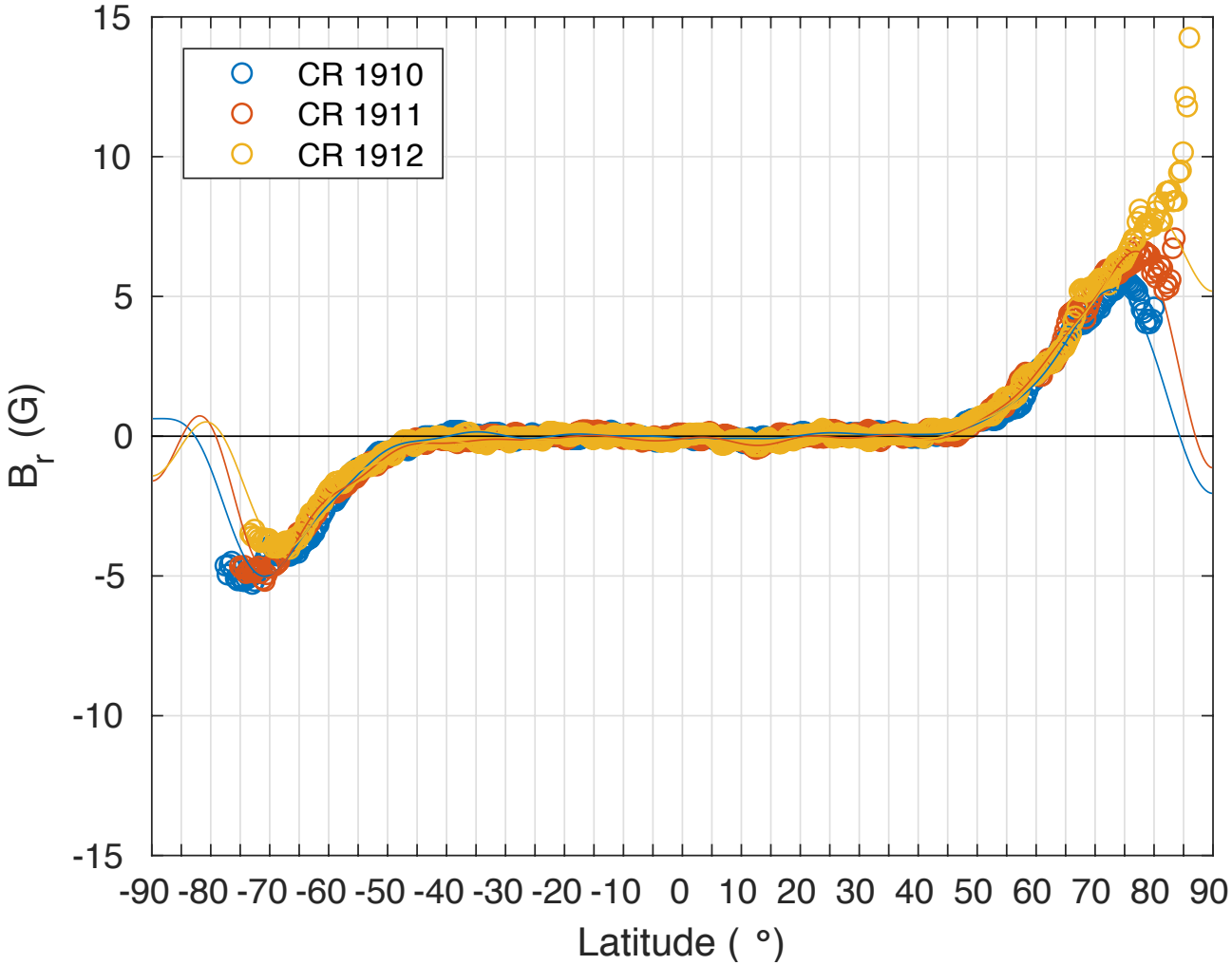
$$g'^{0}_{l} = \frac{\pi}{2} \frac{2l+1}{N_\theta(l+1)} \sum_{i=1}^{N_\theta} \langle B_i^{\mathrm{los}} \rangle P_l^0(\cos\theta_i),$$

where $\langle B_i^{los} \rangle$ is the mean line-of-sight magnetic field at the colatitude of θ_i.

The latitudinal profiles of the radial magnetic field can be reconstructed from the first 24 zonal Gauss coefficients as follows:

$$B_{\mathrm{r}}(\theta) = \sum_{l=1}^{24} (l+1) g'^{0}_{l} P_l^0(\cos\theta)$$

Latitudinal Profiles of the Radial Magnetic Field for Three Consecutive Carrington Rotations

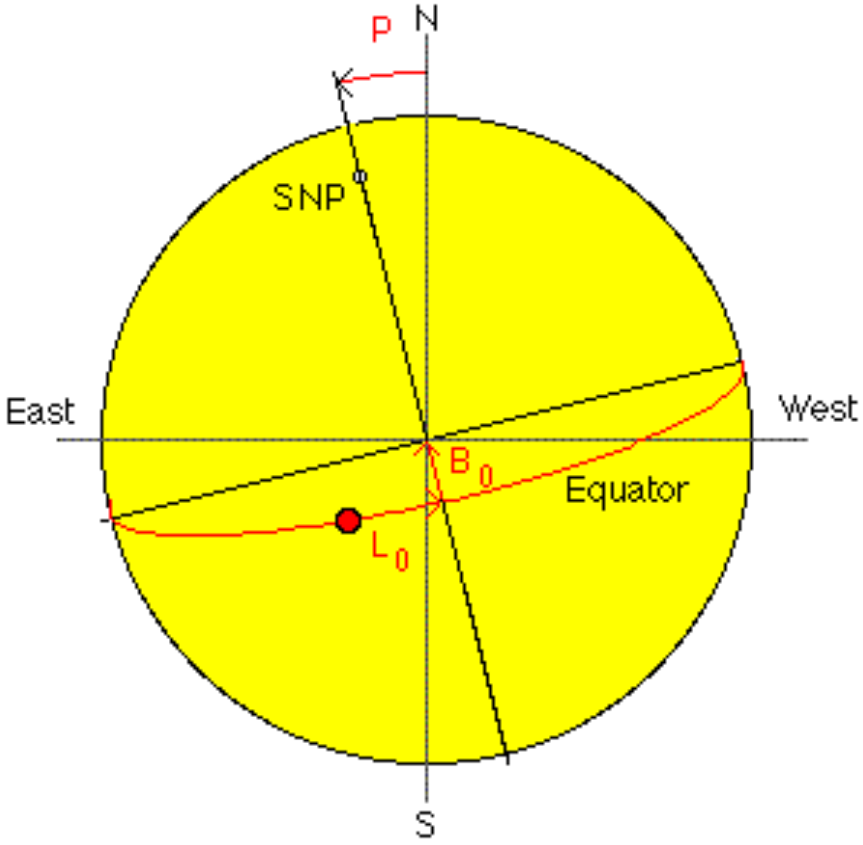


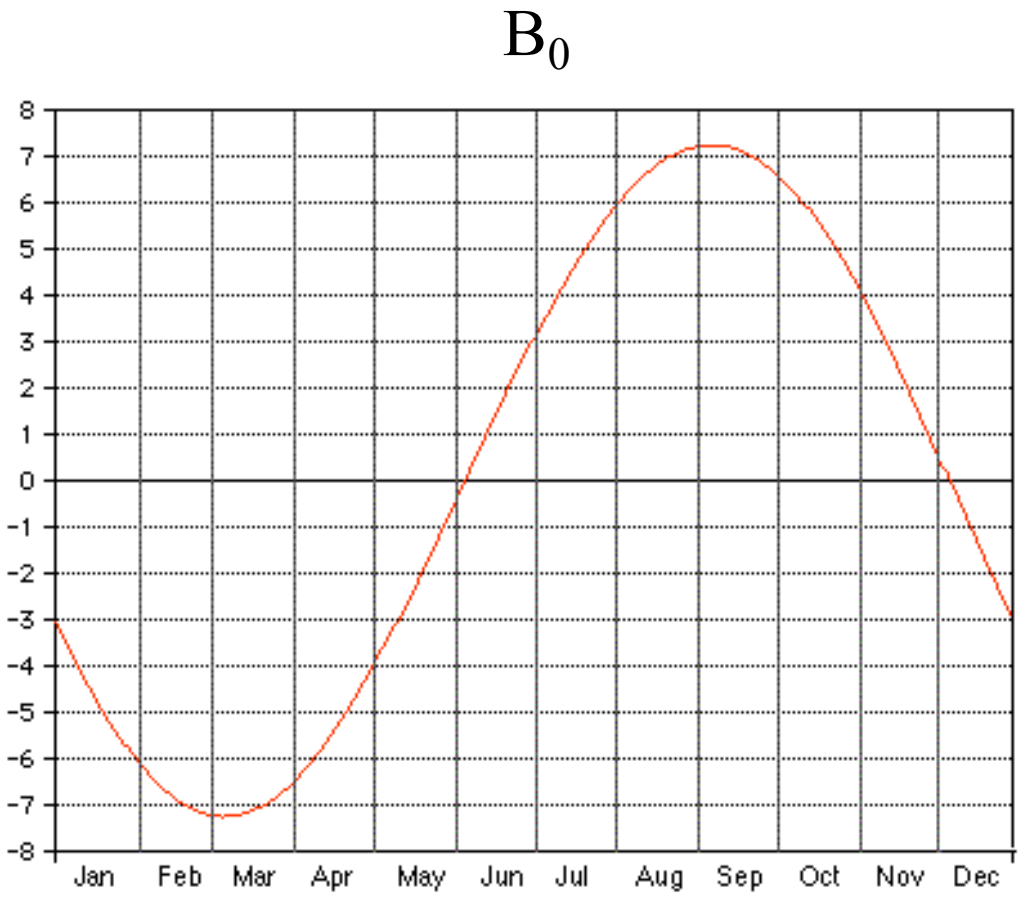
The decline of the polar field at the northern and southern ends of the profiles is an artefact due to large observational errors close to the edge of the visible solar disk.

The change in the latitude range of data is caused by the vantage point effect.

Latitudinal profiles reconstructed from the first 24 harmonic coefficients are shown as solid lines.

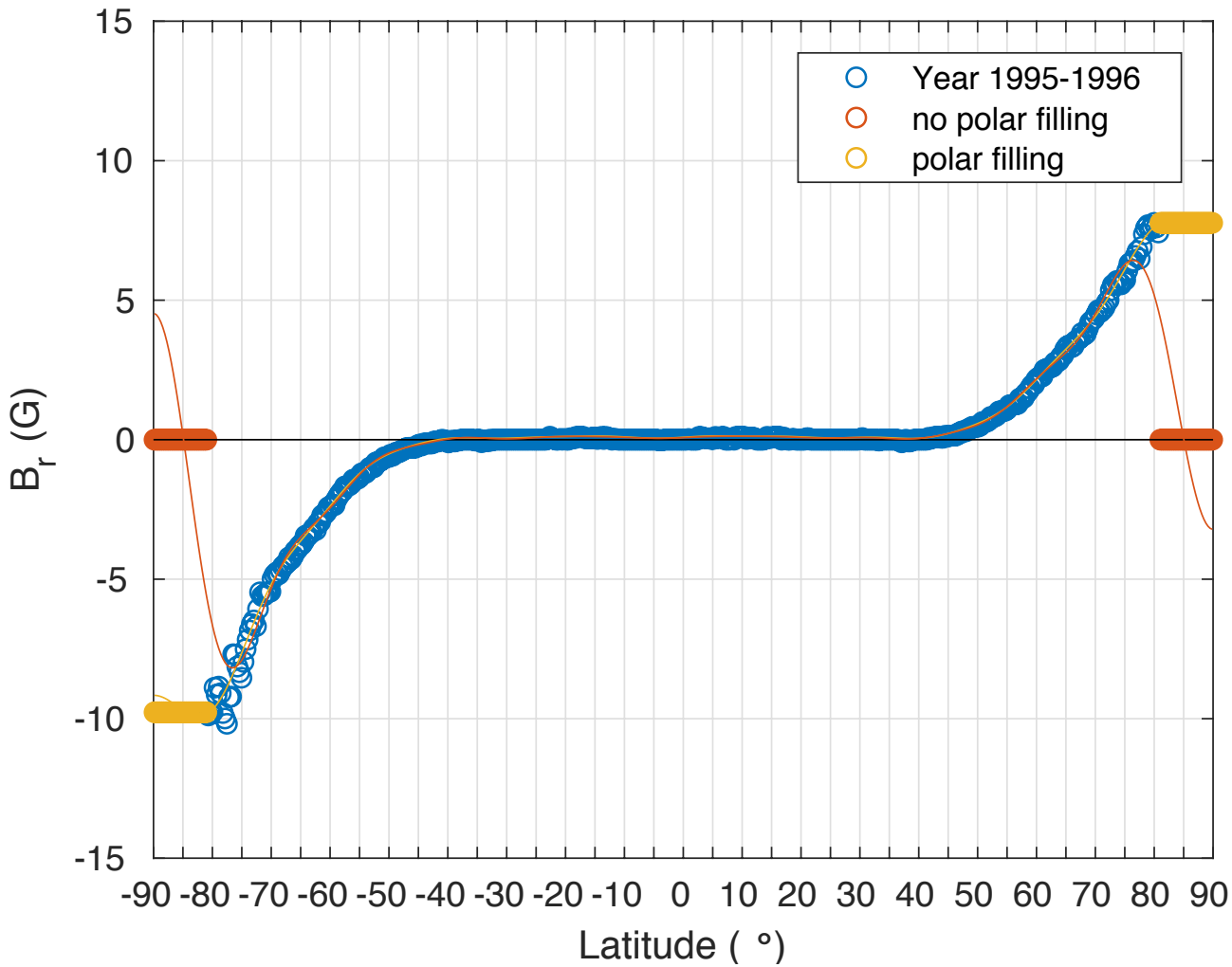
Vantage Point (B_0) Effect in Solar Observations





B_0 is the heliographic latitude of the central point of the solar disk due to the tilt of the ecliptic with respect to the solar equatorial plane.

Latitudinal Profile of the Radial Magnetic Field During Solar Minimum (1995-1996)



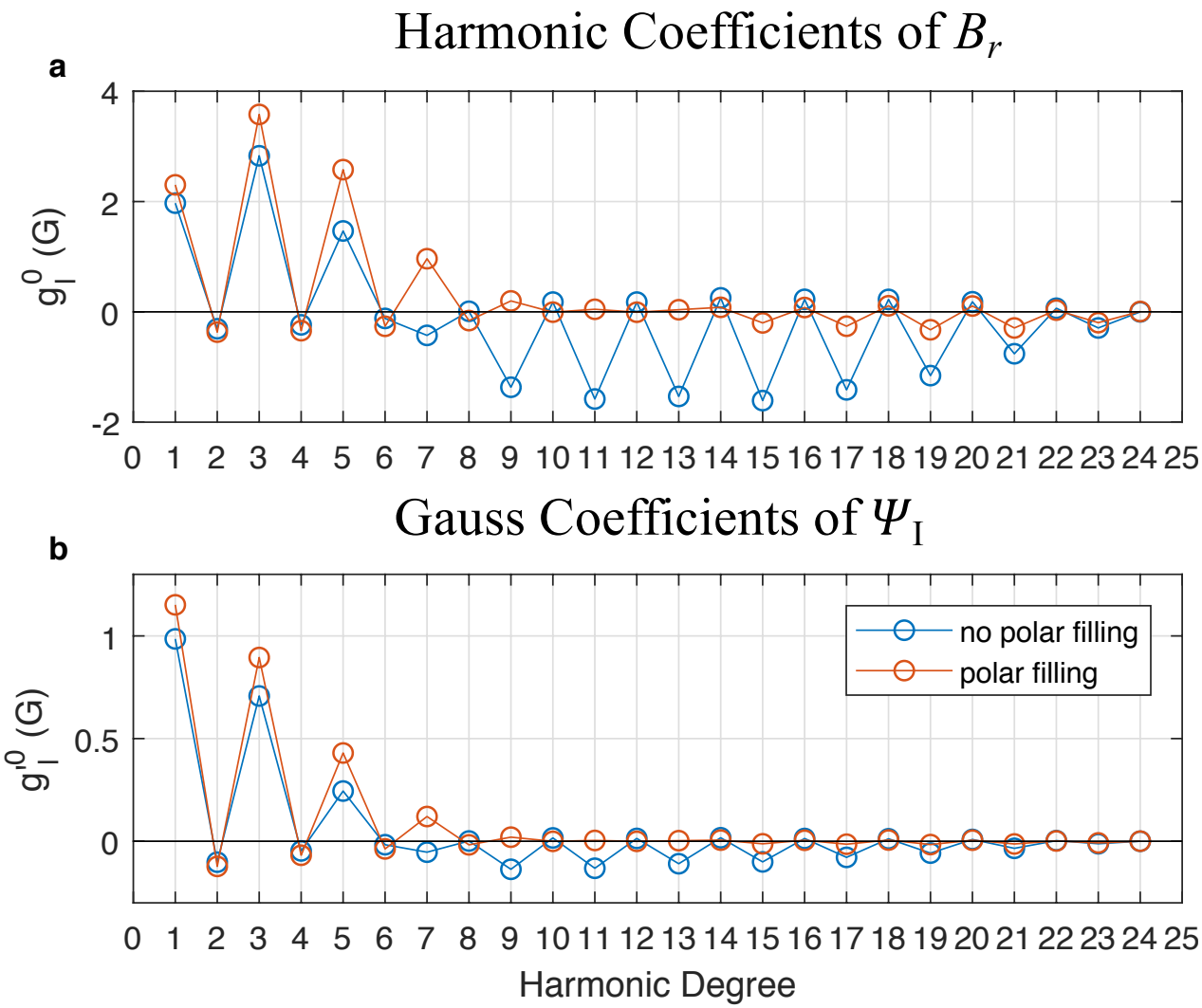
Two-year median profile of the radial magnetic field after removing the erroneous observations at the highest 5º of latitude.

The polar data gaps are filled with zeros (red) and a constant value (yellow), respectively .

The southern polar field is significantly stronger than the northern polar field.

Zieger et al., A&A, 2019

Zonal Harmonic Coefficients Calculated With and Without Polar Filling



Zieger et al., A&A, 2019

Definition of Spatial Power Spectrum

$$\frac{1}{4\pi}\int \mathbf{B}^2 \mathrm{d}\Omega = \frac{1}{4\pi}\int (-\nabla\Psi_{\mathrm{I}})^2 \mathrm{d}\Omega =$$

$$\sum_{l=1}^{\infty}(l+1)\left(\frac{R_0}{r}\right)^{2l+4}\sum_{m=0}^{l}\left[(\mathrm{g}'^{m}_{l})^2 + (h'^{m}_{l})^2\right]$$

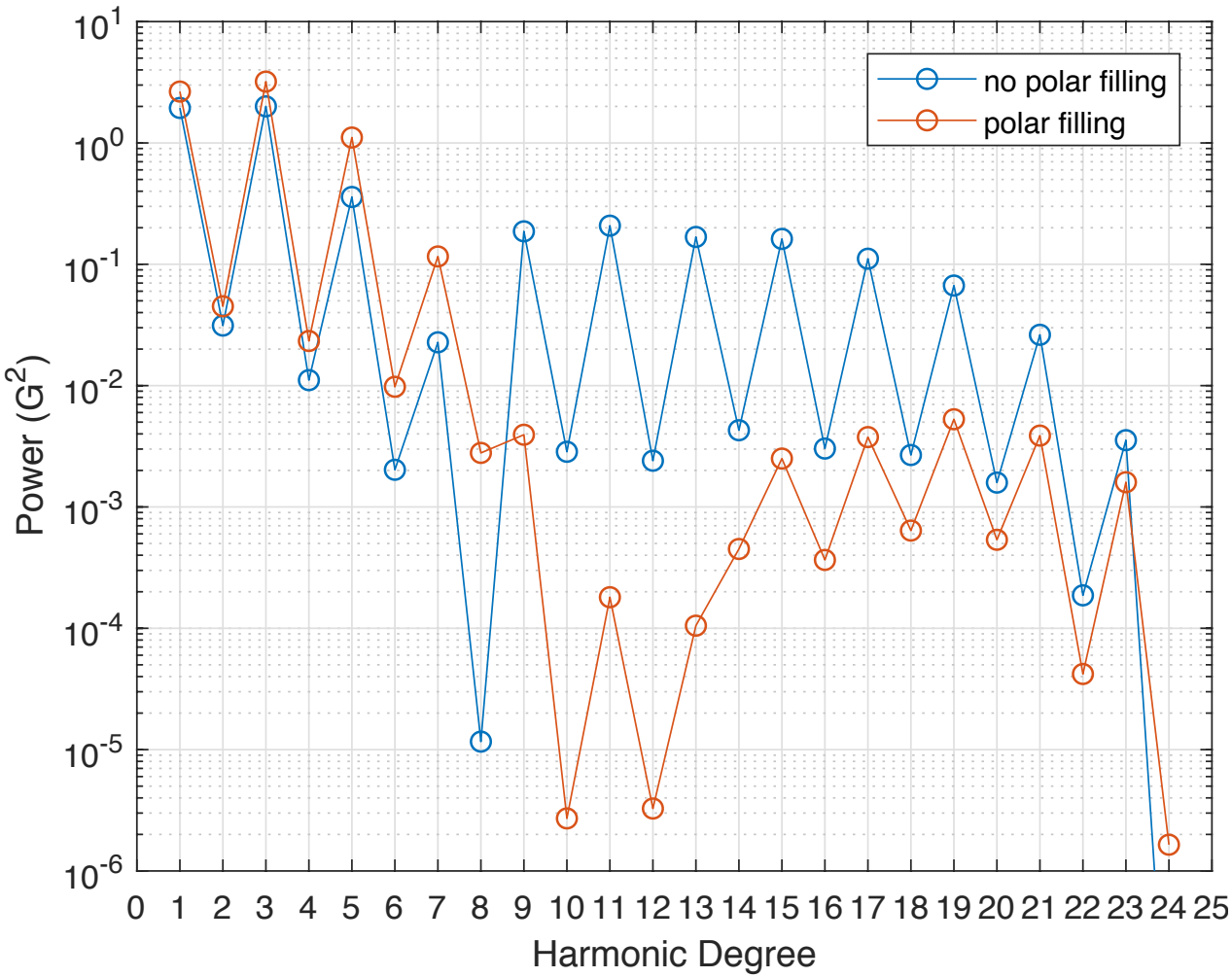
In the photosphere ($r = R_0$) the power-per-degree spectrum is

$$S_l^{\mathrm{degree}} = (l+1)\sum_{m=0}^{l}\left[(\mathrm{g}'^{m}_{l})^2 + (h'^{m}_{l})^2\right]$$

and the zonal ($m = 0$) spatial power spectrum becomes

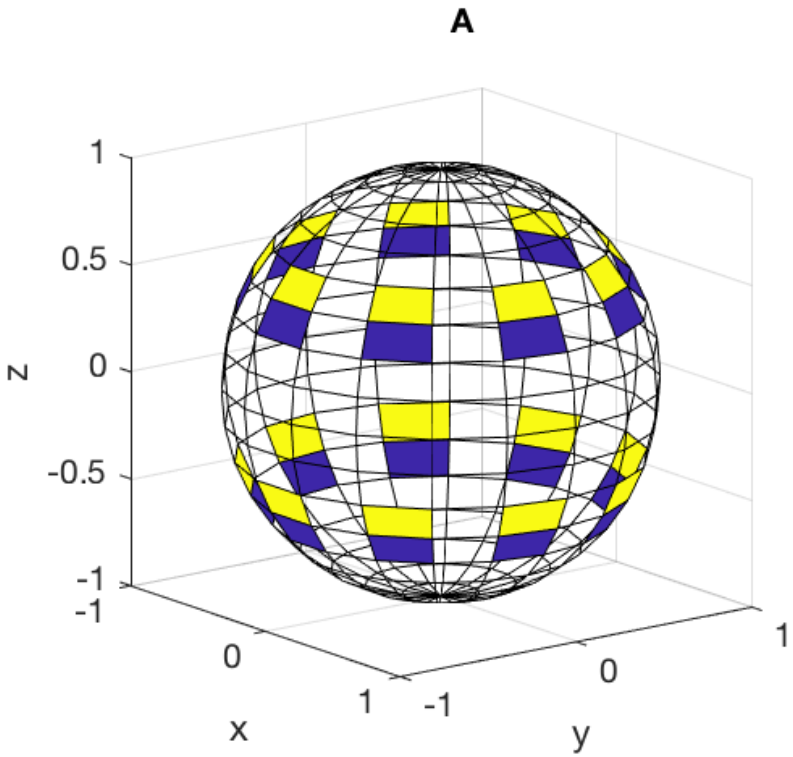
$$S_l^{\mathrm{zonal}} = (l+1)(\mathrm{g}'^{0}_{l})^2 = \frac{1}{(l+1)}(\mathrm{g}^{0}_{l})^2$$

Zonal Spatial Power Spectrum of the Photospheric Magnetic Field (1995-1996)

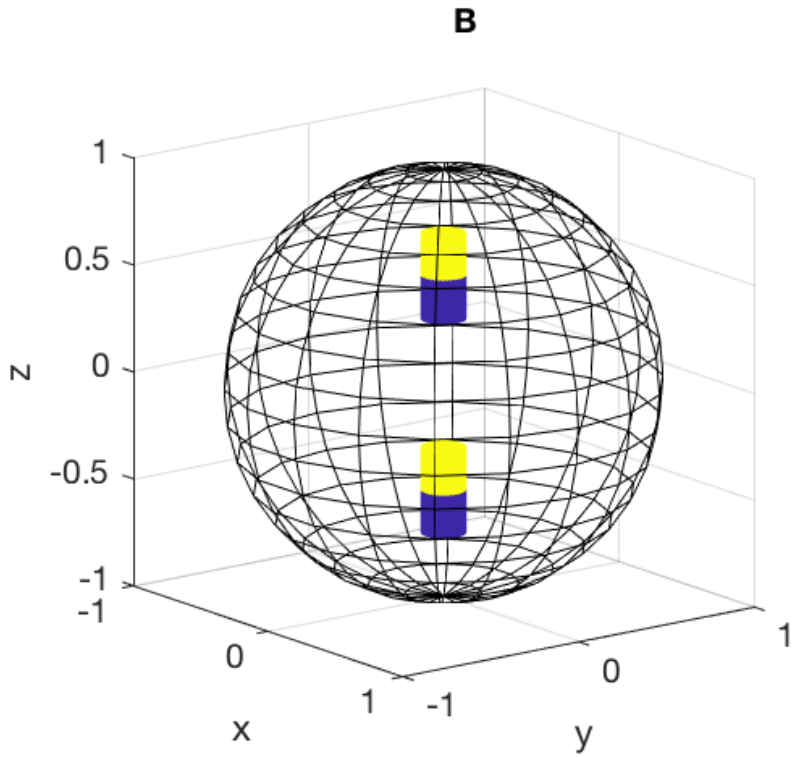


Zieger et al., A&A, 2019

Two-Dipole Model of the Photospheric Magnetic Field



Two rings of dipoles representing the north-south magnetic component of decaying active regions in the photosphere in an axisymmetric case.



Two axial dipoles placed at the center of each dipole ring in panel **A**. The magnetic potential of the dipoles in panel **A** and **B** are nearly identical at the solar surface.

Fitting the Two-Dipole Model to the Observed Zonal Gauss Coefficients

Theoretically derived zonal Gauss coefficients of two eccentric axial dipoles

$$g_l^0 = l a_1 z_1^{(l-1)} + l a_2 z_2^{(l-1)}$$

where a_1 and a_2 are the strength of the two dipoles and z_1 and z_2 are their locations along the z-axis of symmetry.

The four unknown parameters of the two-dipole model can be exactly solved using the equations for the first four Gauss coefficients:

$$z_1 = (9g_1^0 g_4^0 - 6g_2^0 g_3^0 + \sqrt{3}(27(g_1^0)^2(g_4^0)^2 - 108 g_1^0 g_2^0 g_3^0 g_4^0 + 64 g_1^0 (g_3^0)^3 + 54(g_2^0)^3 g_4^0 -$$

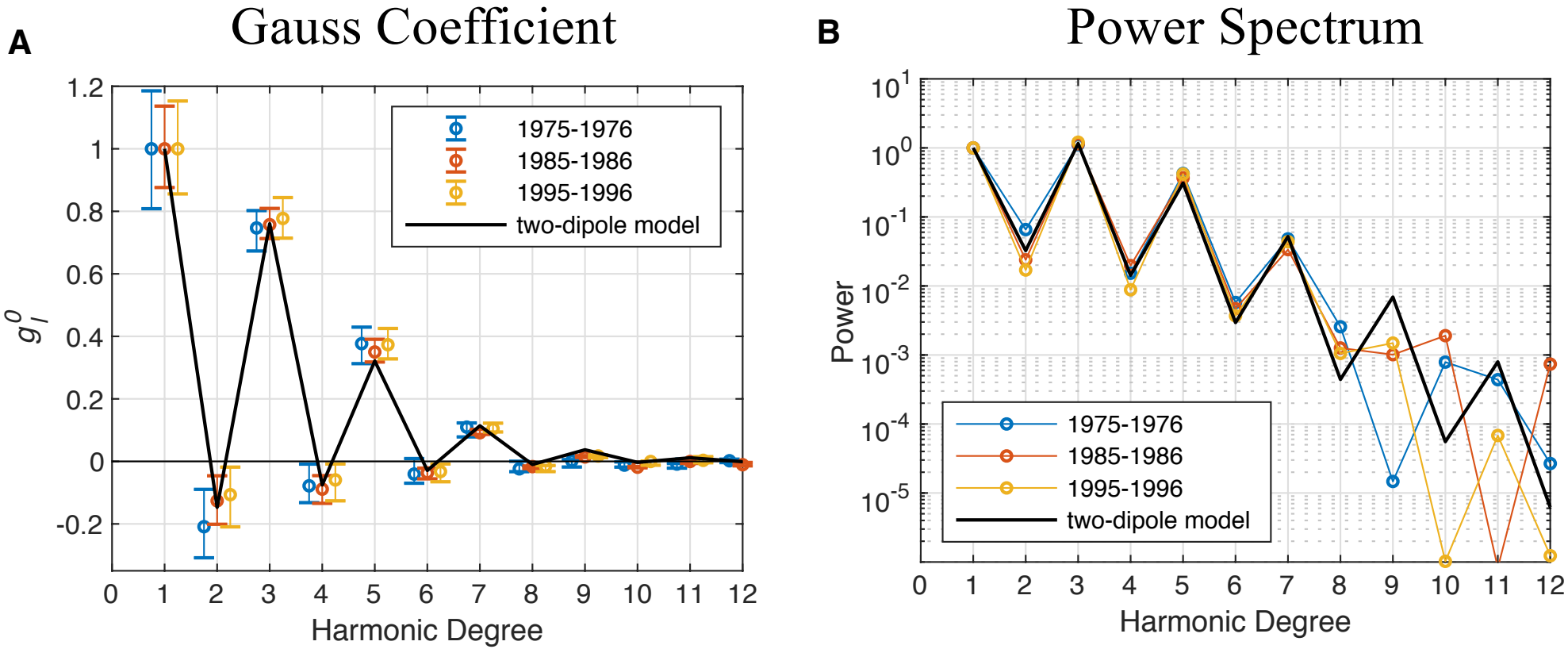
$$36(g_2^0)^2(g_3^0)^2)^{\frac{1}{2}})/(24 g_1^0 g_3^0 - 18(g_2^0)^2),$$

$$z_2 = (2g_3^0 - 3g_2^0\, z_1)/(3g_2^0 - 6g_1^0 z_1),$$

$$a_1 = (g_2^0 - 2g_1^0 z_2)/(2z_1 - 2z_2),$$

$$a_2 = g_1^0 - a_1.$$

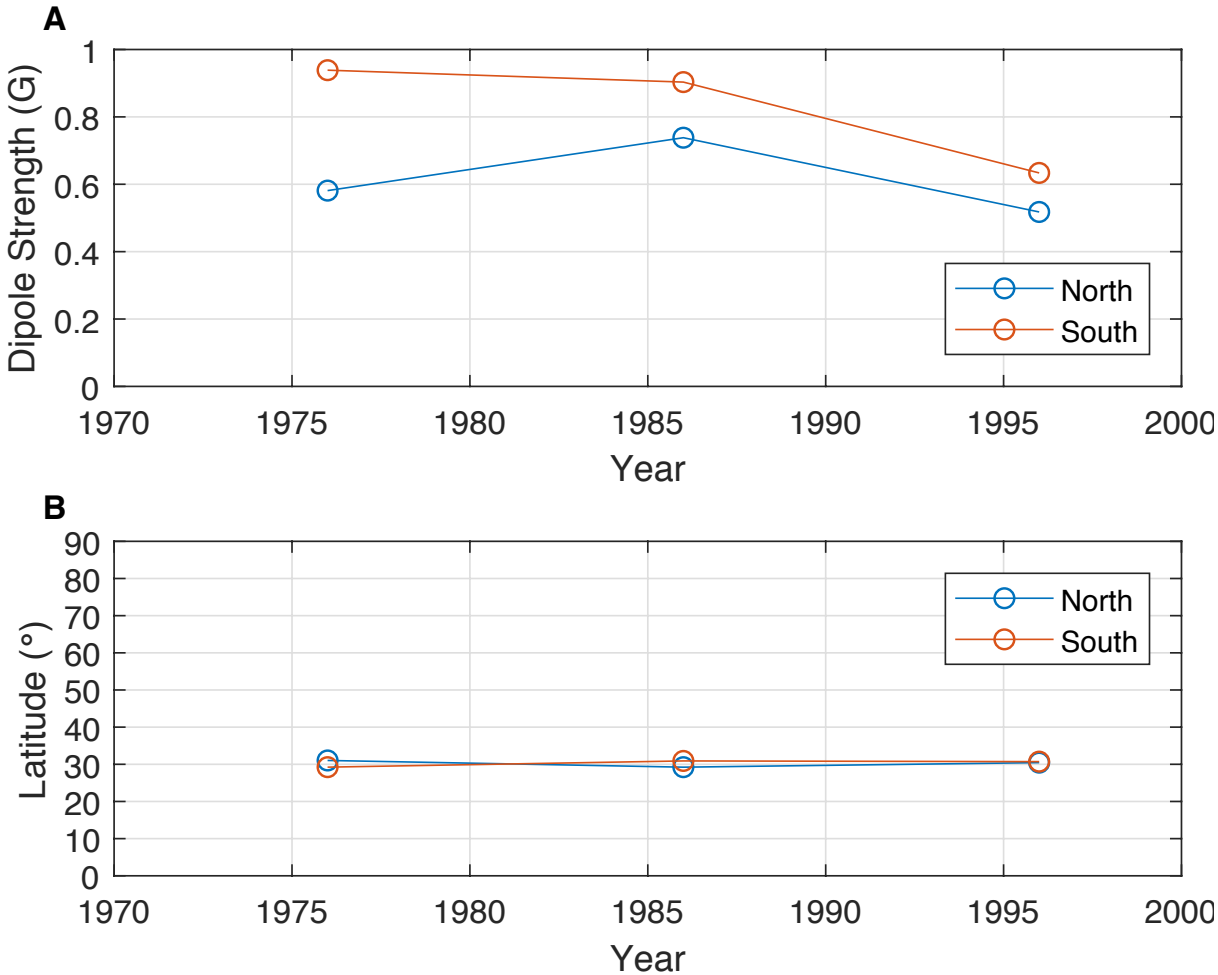
Normalized Zonal Gauss Coefficients and Spatial Power Spectra for Three Solar Minima



The two-dipole model (black) fitted to the first four zonal Gauss coefficients can reproduce the observed spatial structure of the photospheric magnetic field up to the harmonic degree 8.

The low-order even zonal Gauss coefficients are significantly different from zero, indicating a persistent north-south asymmetry during solar minima.

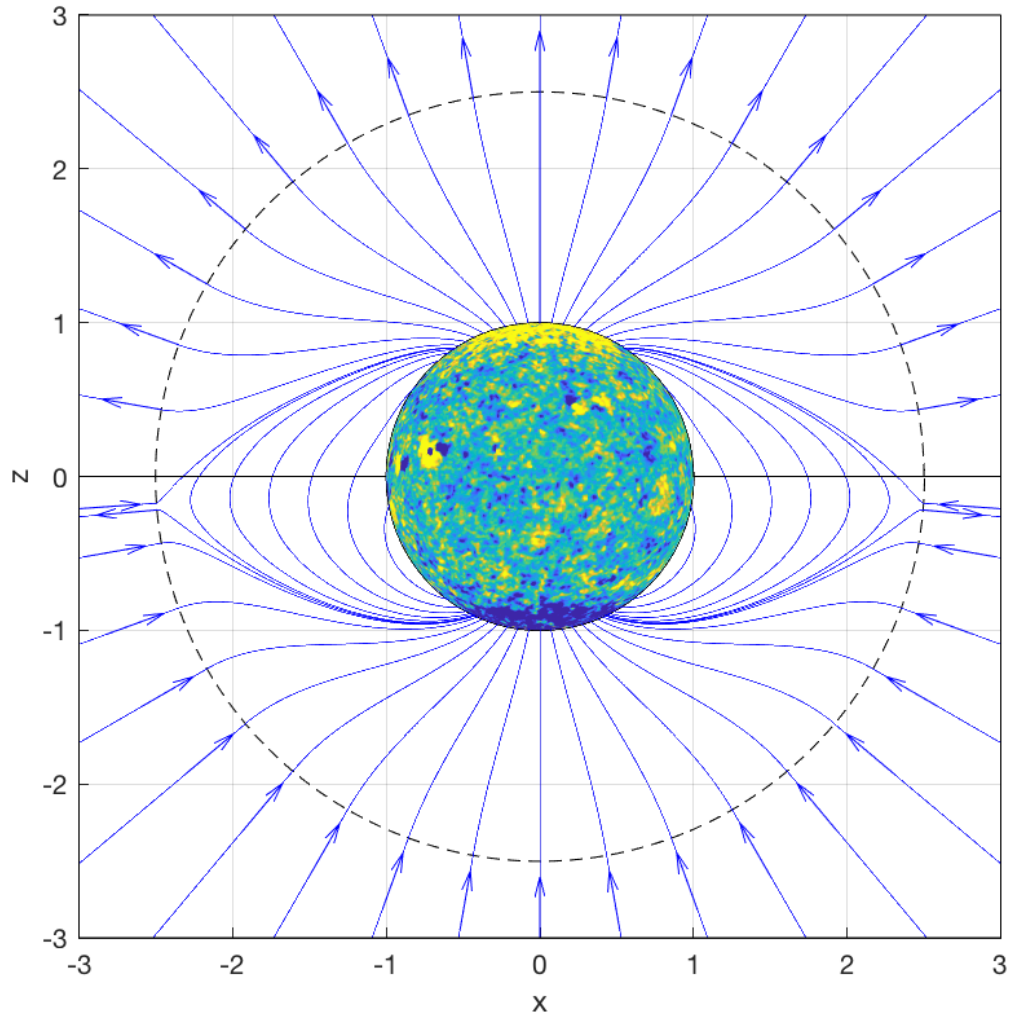
Parameters of the Northern and Southern Dipoles



The southern dipole is stronger than the northern dipole during all the three solar minima.

The northern and southern dipoles are located at similar northern and southern latitudes, implying that the asymmetry is caused by the different dipole strengths.

Coronal Magnetic Field Arising From the Two-Dipole Model of the Photospheric Magnetic Field



The potential field source surface (PFSS), where the coronal magnetic field becomes radial, is marked by a dashed circle. The heliospheric current sheet, where the magnetic field reverses, is tilted towards the south by 4.1º.

Conclusions

- The two-dipole model can reproduce the spatial structure of the photospheric magnetic field during solar minimum up to harmonic degree 8.

- The north-south asymmetry is caused by the different strengths of the northern and southern dipoles rather than the difference in their heliographic latitudes.

- The southern dipole was found to be stronger during all the three solar minima, indicating a persistent north-south asymmetry in the operation of the solar dynamo.

- The photospheric asymmetry results in a southward tilted heliospheric current sheet (3°-5°) during solar minima, which is confirmed by heliospheric observations.

- The two-dipole model could be used to fill in the polar data gaps in synoptic maps of the photospheric magnetic field.

Thank You!