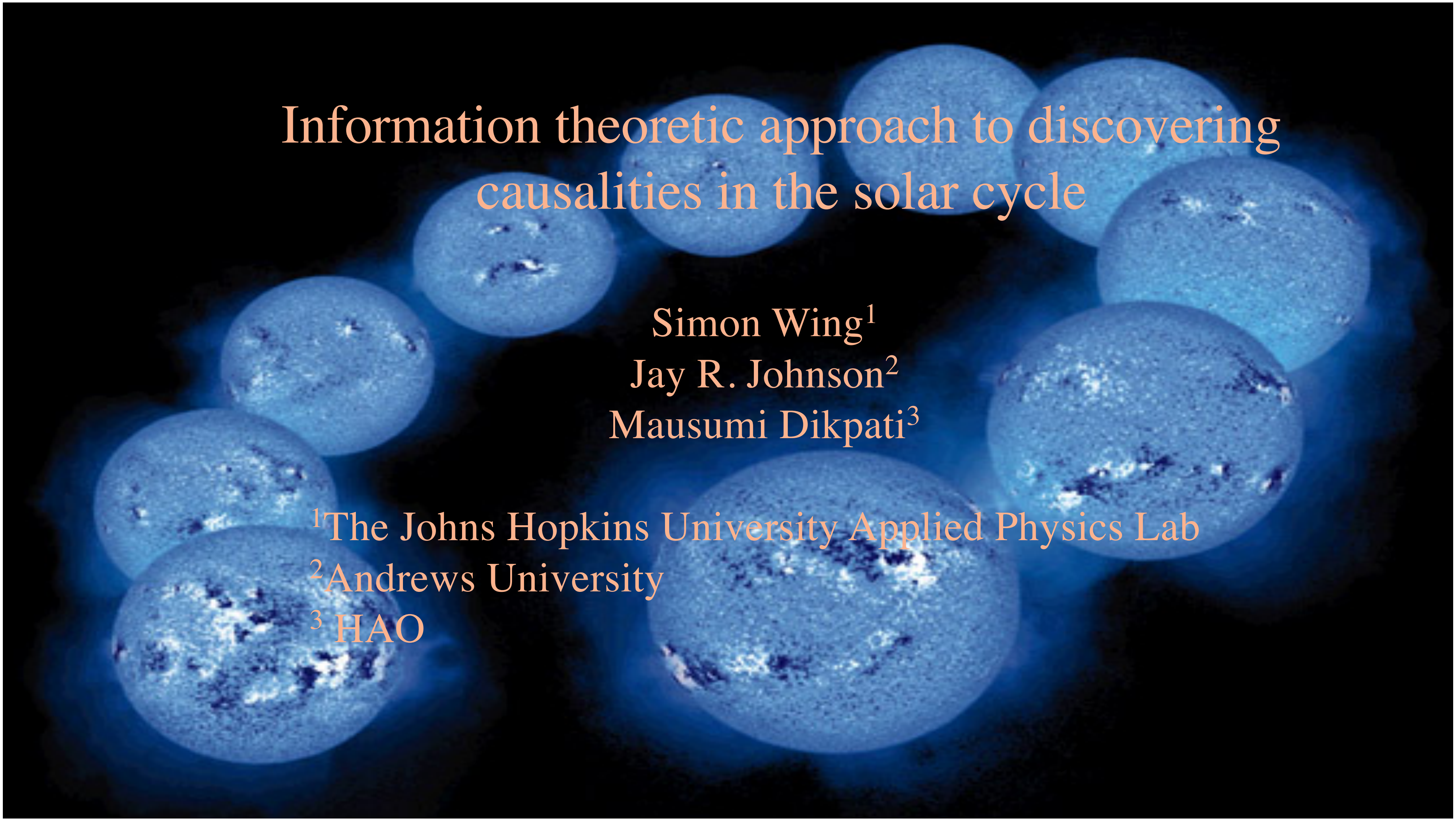

Information theoretic approach to discovering causalities in the solar cycle

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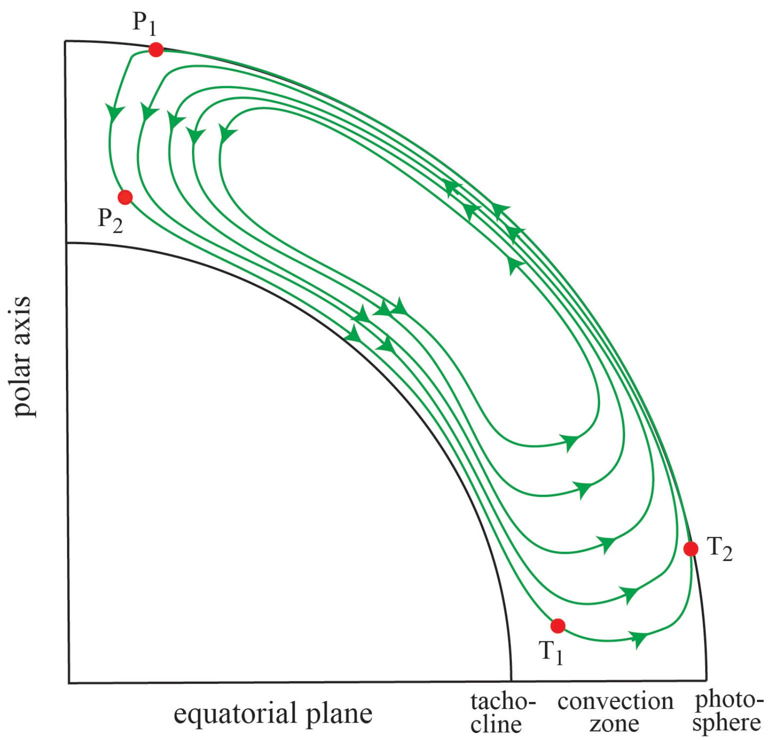
Introduction and motivation

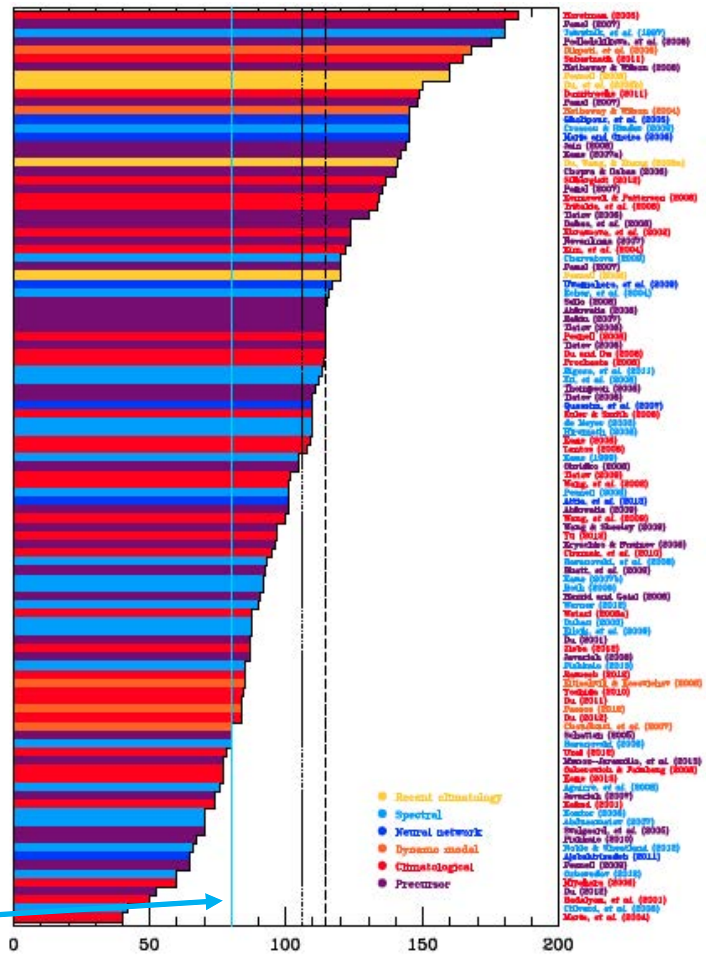


Babcock-Leighton type model
[Babcock, 1961; Leighton, 1964; 1969]

- The solar dynamo is not fully understood
- it is still a challenge to predict SSN
- apply information theory to identify causal parameters and the response lag times
- provide observational constraints to solar cycle models and theories:
  - surface flux transport models [e.g., Devore and Sheeley, 1987; Wang et al., 1989; 2005]
  - flux transport dynamo models [e.g., Dikpati et al., 2006; Choudhuri et al., 2007]
- Babcock-Leighton type model
  - important parameters:
    - meridional flow
    - polar field
    - flux emergence

Model predictions of SSN at solar max for solar cycle 24

Pesnell [2016]



observed International SSN from SIDC

It is still a challenge to predict sunspot number (SSN)

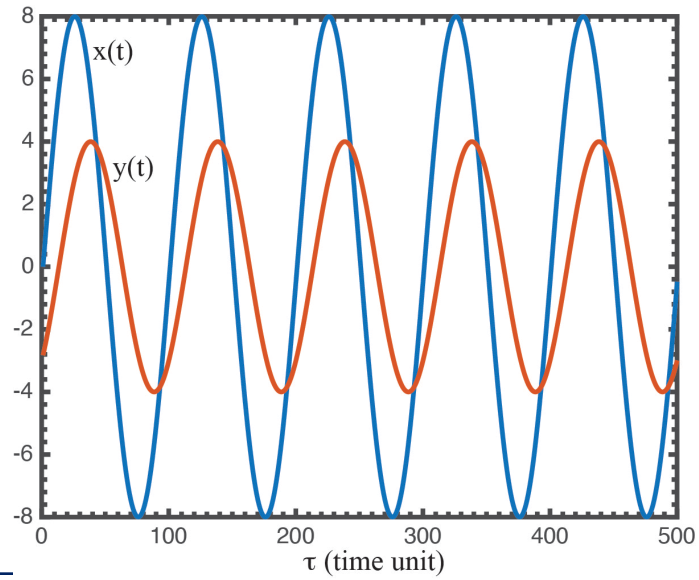
transfer entropy

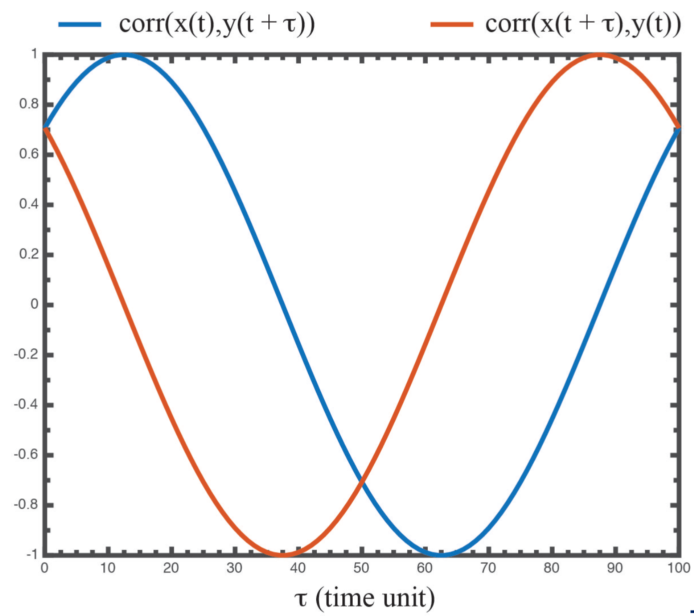
A common method to establish causal-relationships between two time series, e.g., [x_t] and [y_t], is to use a time-shifted cross-correlation function

$$r(\tau) = \frac{\langle x_t \, y_{t+\tau} \rangle - \langle x \rangle \langle y \rangle}{\sqrt{\langle x^2 \rangle - \langle x \rangle^2} \, \sqrt{\langle y^2 \rangle - \langle y \rangle^2}}$$

where r = correlation coefficient and τ = lag time

The results may not be clear if x and y have multiple peaks





transfer entropy

A better alternative is to use transfer entropy [Schreiber, 2000]

$$TE_{x \to y}(\tau) = \sum_t p(y_{t+\tau}, yp_t,\ x_t)\ \log\left(\frac{p(y_{t+\tau} \mid yp_t,\ \ x_t)}{p(y_{t+\tau} \mid yp_t)}\right)$$

$yp_t = [y_t,\ y_{t-\Delta}, \cdots, y_{t-k\Delta}]$, $k + 1$ = dimensionality of the system, and Δ = first minimum in MI

TE($x \to y$) gives a measure of information transfer from x to y given that the past values of y are known

TE can be considered a special case of conditional mutual information (CMI)

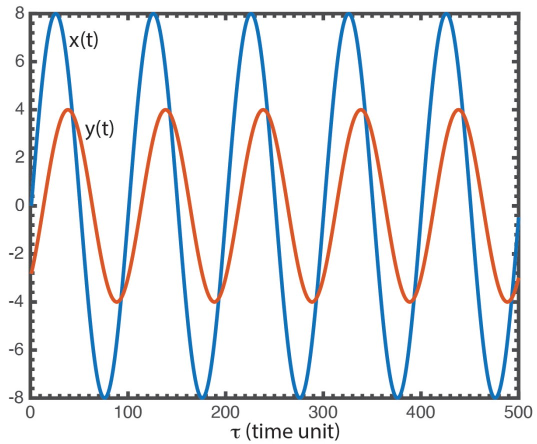
$$TE_{x \to y}(\tau) = CMI(y(t + \tau), x(t) | yp(t))$$

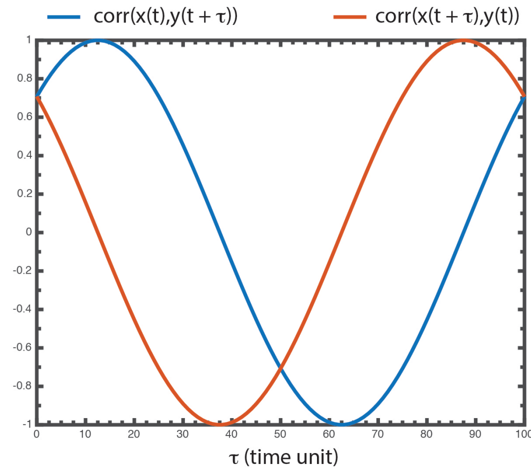
if no information flow from x to y, TE($x \to y$) = 0

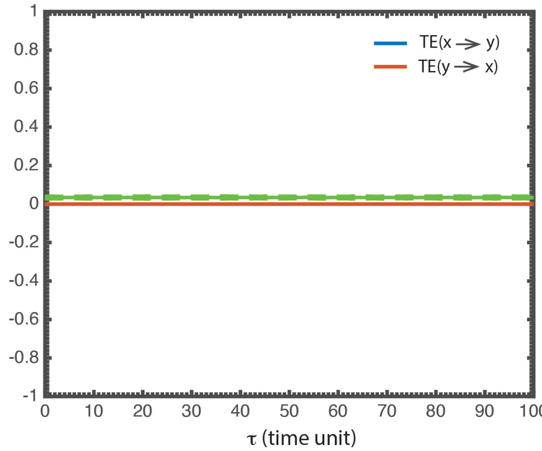
unlike correlation, TE($x \to y$) ≠ TE($y \to x$)

TE can take into account nonlinearities in the system

Transfer entropy





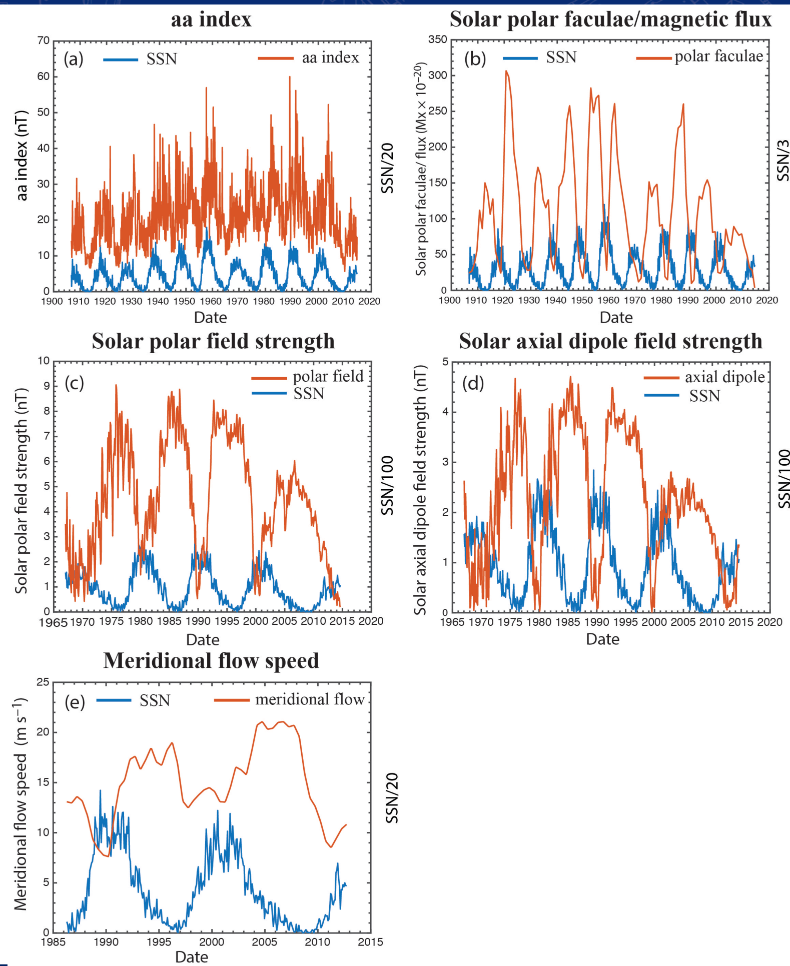


Data set

- SSN 1749–2016 – SILSO website in Belgium
- sunspot area1874–2016 – NASA MSFC website
- meridional flow 1986–2012 – MWO [Ulrich, 2010] (from R. Ulrich)
- polar faculae 1906–2014 – MWO, WSO, SOHO [Munoz-Jaramillo et al., 2012] at Solar Polar Fields Dataverse website
- polar field 1967–2015 – MWO, SWO [Ulrich, 1992; Wang and Sheeley, 1995] (from Y.-M. Wang)
- axial dipole strength 1967–2015 – MWO, WSO [Wang and Sheeley, 1995; 2009] (from Y.-M. Wang)
- aa index 1868–2010 – NOAA NCEI website

All data are evaluated (averaged, interpolated) at monthly resolution

Solar cycle variations in solar data

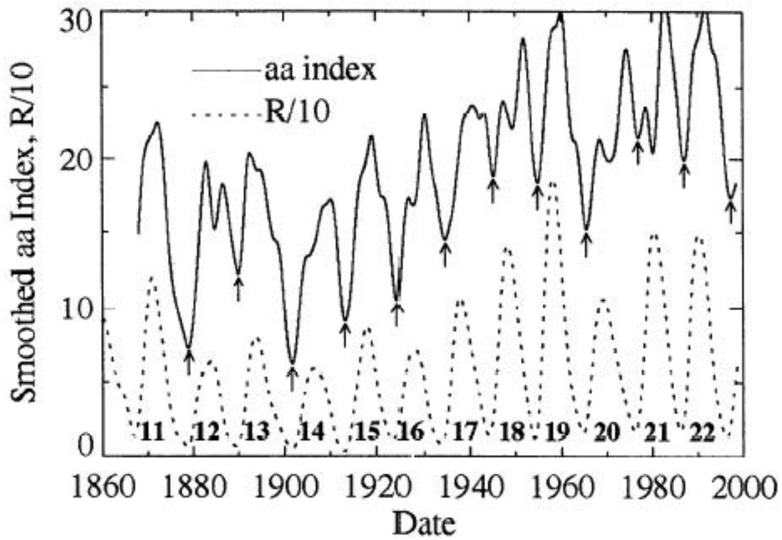


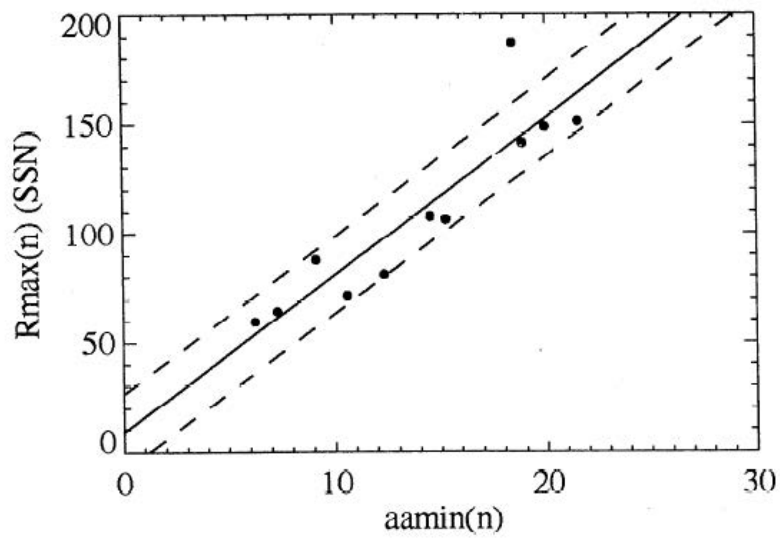
Solar cycle variations of

- aa index
- polar faculae
- polar field
- axial dipole field strength
- meridional flow
- Sunspot number (SSN)

SSN and aa index

Hathaway et al. [1999]



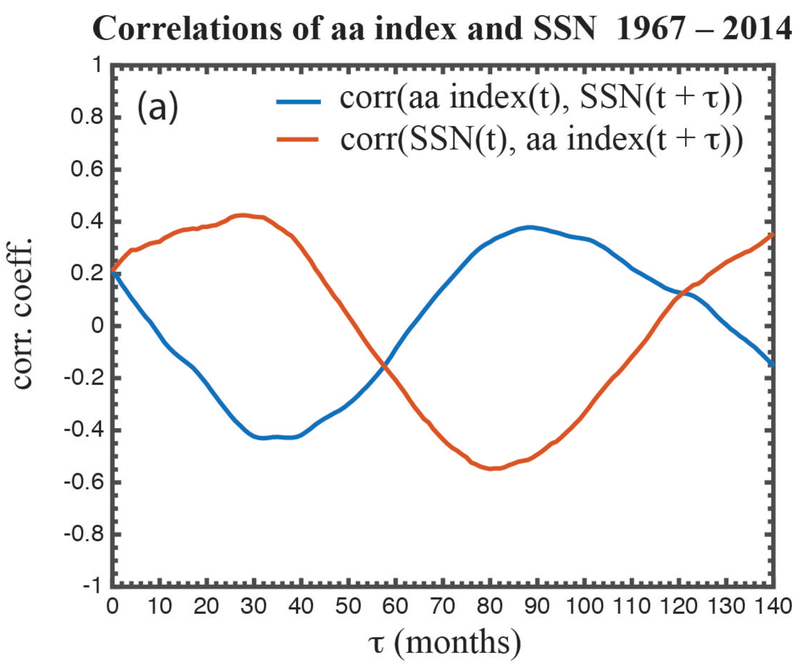


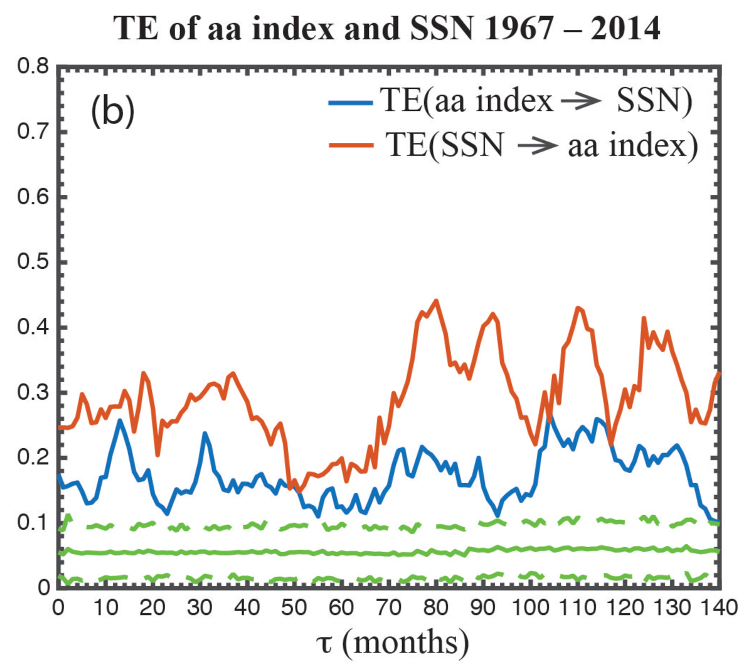
Both, SSN and aa index exhibit cyclical variations

SSN→ aa index

aa index→ SSN [e.g., Ohl, 1966; Hathaway et al., 1999; Schatten and Pesnell, 1993; Wang and Sheeley, 2009]

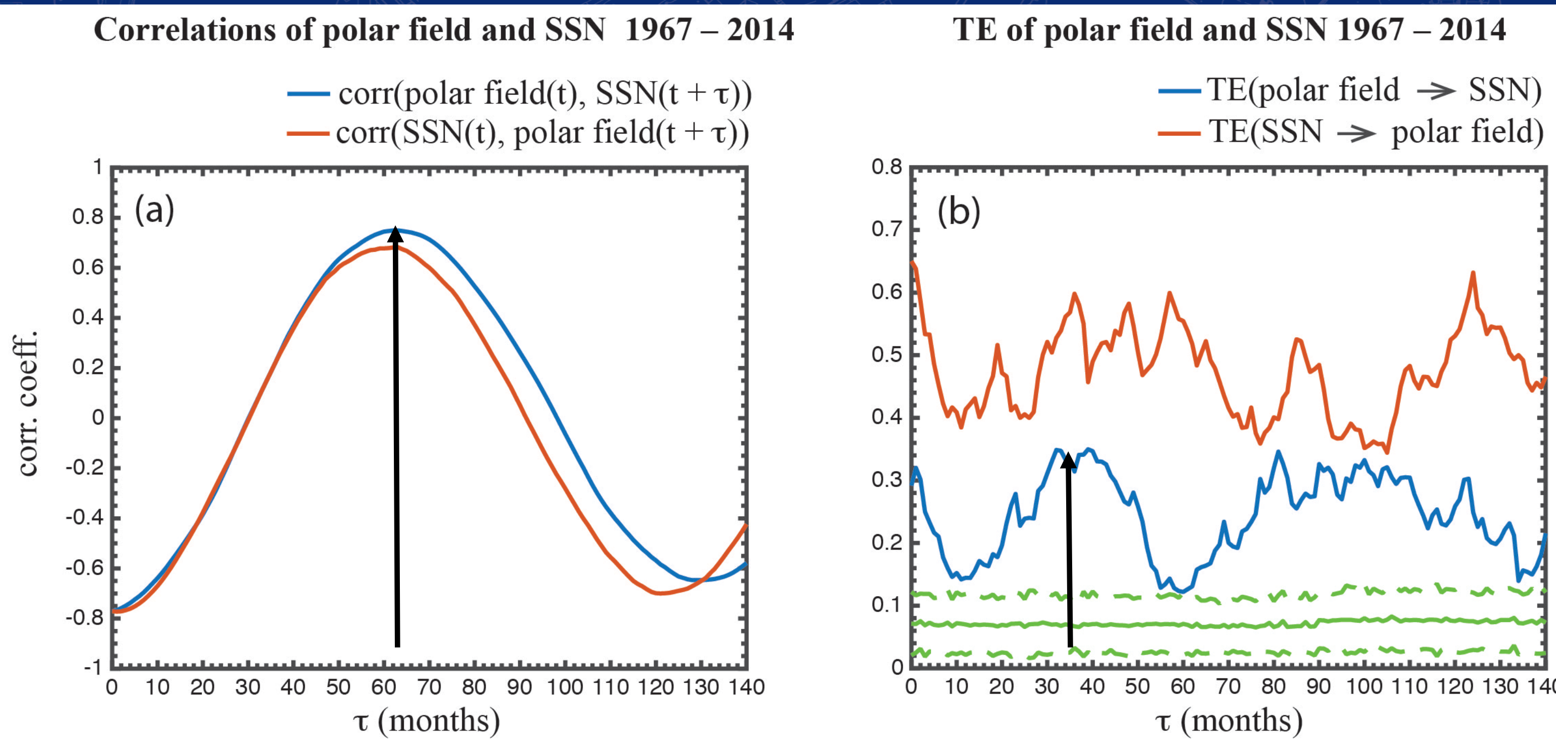
SSN and aa index





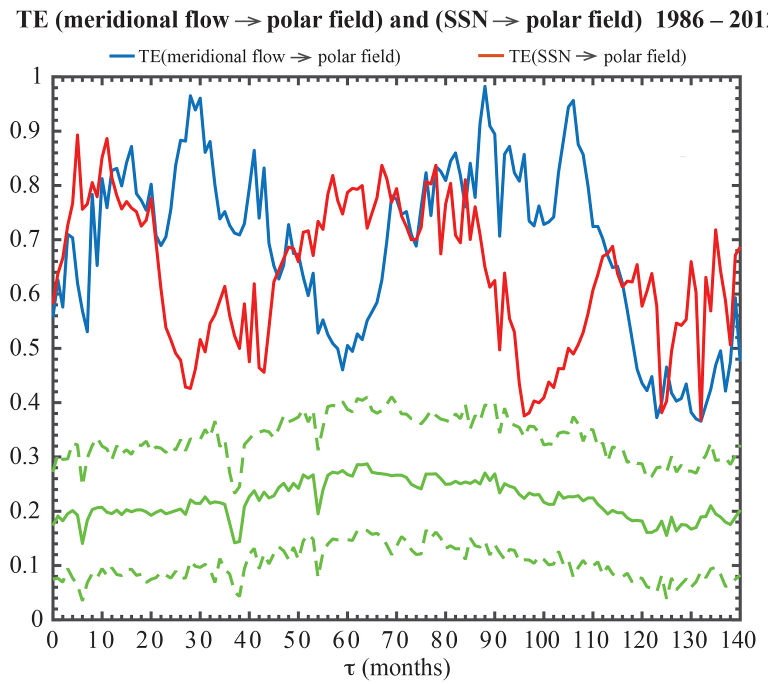
- peak |corr(aa index(t), SSN(t + τ))| ~ peak |corr(SSN(t), aa index(t + τ))|
- **But,** TE(SSN→aa index) > TE(aa index → SSN)
- more information is transferred from SSN to aa index than the other way around; aa index is a poor proxy for the solar polar field – this information cannot be obtained from correlational analysis

SSN and solar polar field



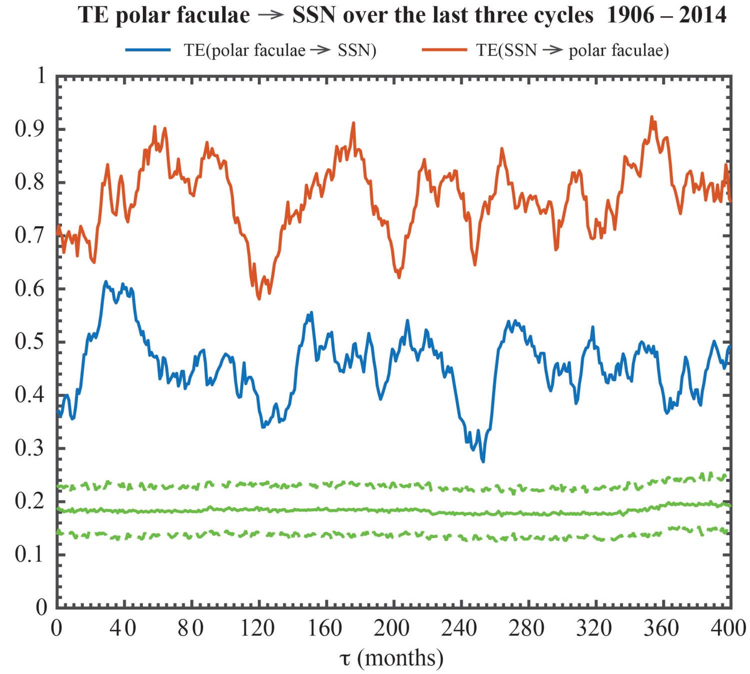
- TE(polar field → SSN) peaks around τ ~30–40 months, not 66 months assumed in some models
- peak significance = (peak TE – mean(noise))/σ(noise) = 19 σ
- TE(SSN→ polar field) is significant [Upton and Hathaway, 2014]

Which parameters control the polar field?



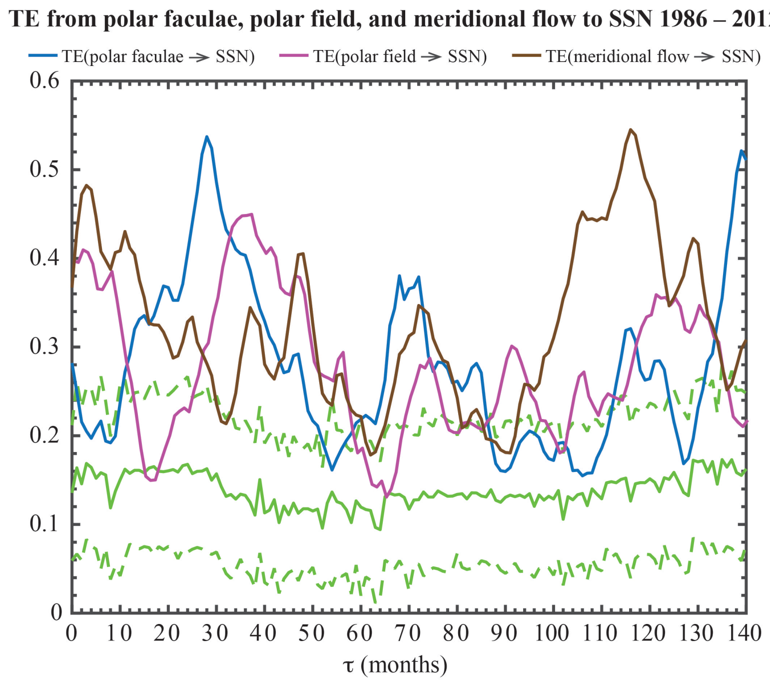
- surface flux transport models [e.g., Devore and Sheeley, 1987; Wang et al., 1989;2005] and flux transport dynamo models [e.g., Dikpati et al., 2006, Choudhuri et al., 2007]: meridional flow controls the strength of the polar field
- amount flux emergence (SSN) controls the polar field [Upton and Hathaway, 2014]
- TE(meridional flow→polar field) peaks τ~30-40 (pos corr),~90-110 months (neg corr)
- TE(SSN→ polar field) peaks τ ~50–80 months (pos corr)

Are the past n cycles important for predicting SSN?



- Dikpati et al. [2004] suggested that meridional flow is slower at the bottom of the convection zone and hence the polar fields from the last 3 cycles should affect SSN (see also Charbonneau & Dikpati, 2000)
- TE(polar faculae → SSN) peaks at τ ~30-40 months but persists at a lower level thereafter for at least 400 months (~ 3 solar cycles)
- There are minima at τ ~1 and ~2 solar cycle periods

Information transfer from polar field, polar faculae,and meridional flow to SSN

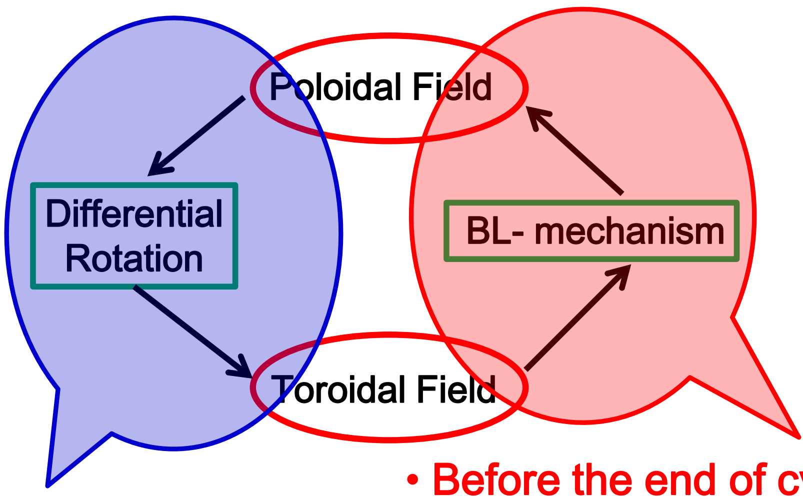


- noisy because data have shorter timespan, limited by meridional flow data
- TE([polar faculae, polar field]→SSN) > TE(meridional flow → SSN) at τ ~30–40 months, which may be consistent with Dikpati et al. [2010] model.
- TE(meridional flow → SSN) peaks around τ ~120 months (~1 solar cycle period), suggesting the meridional flow can be used to predict SSN one solar cycle period ahead

Conversion from toroidal to poloidal field is hard

Jie Jiang (Space Climate 7, 2019)

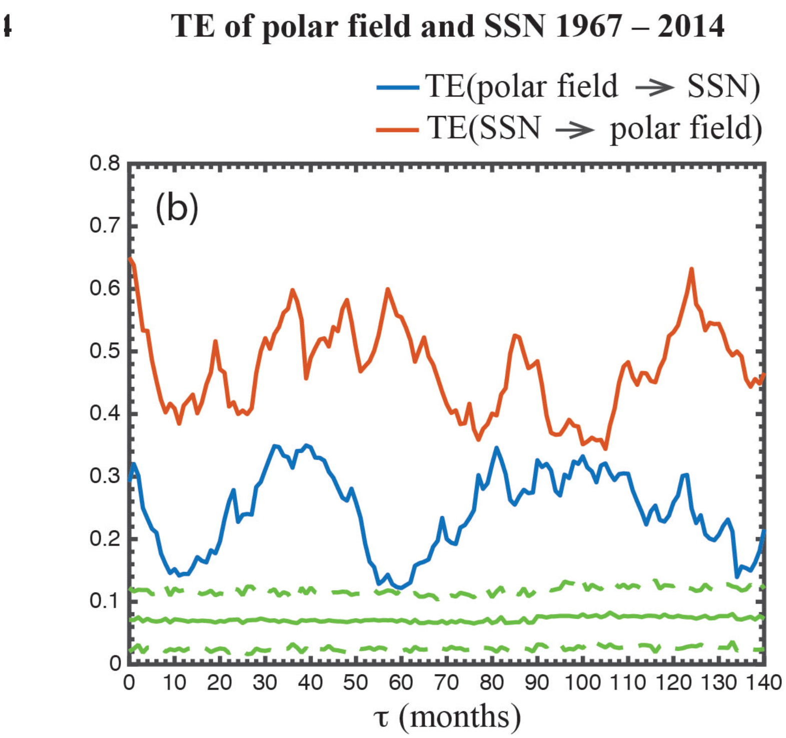
Sketch of model-based predictions



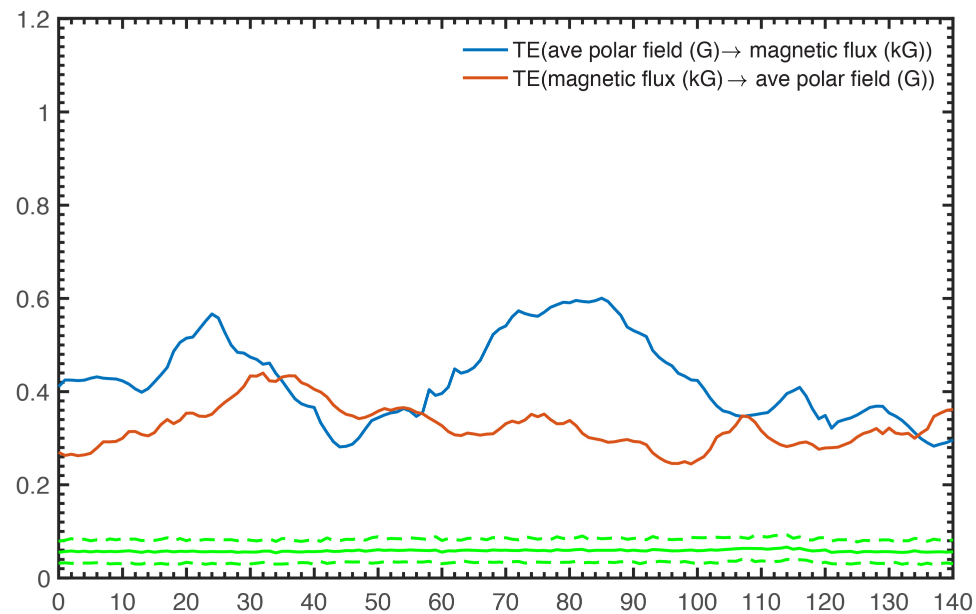
- Around the end of cy. 23
- Flux transport dynamo-based prediction
- Easy part –linear

- Before the end of cy.24
- Surface flux transport model-based prediction
- Hard part –randomness & nonlinearities

Comparing observations with Dikpati simulation



Dikpati simulation



Summary and Conclusion

- TE(SSN → aa index) > TE(aa index → SSN)
- TE(polar field → SSN) peaks at $\tau \sim$ 30–40 months (the response of SSN to polar field peaks ~3-4 years, not 5.5 years).
- Polar faculae transfers similar amount of information as polar field to SSN, confirming that polar faculae is a good proxy of polar field
- TE(polar field → SSN) > TE(meridional flow → SSN) at τ ~30–40 months
- TE(meridional flow → SSN) peaks at τ ~120 months
- TE(meridional flow → polar field) peaks at τ ~30 (pos corr), ~90–110 months (neg corr) whereas TE(SSN→ polar field) peaks τ ~50–80 months (pos corr)
- TE(polar faculae → SSN) peaks at τ ~30–40 months, but persists at lower level for at least 3 solar cycles
- Our results provide observational constraints to solar cycle models and theories
- More work to be done to understand our results
- Interested to try other models or parameters