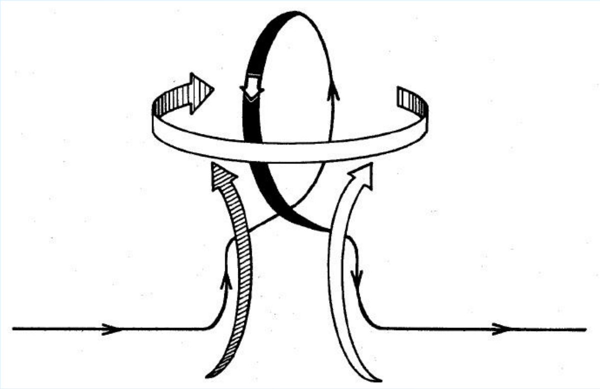
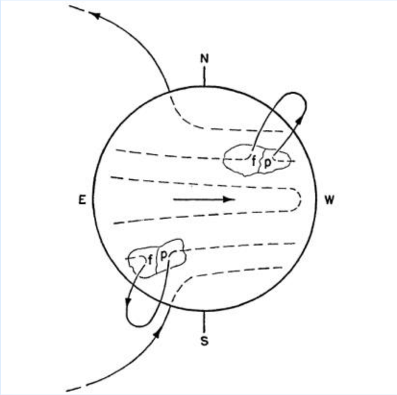
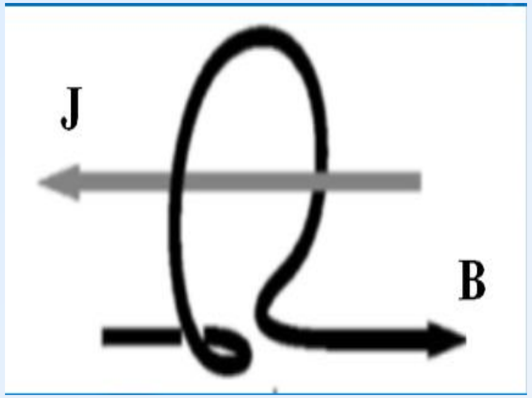
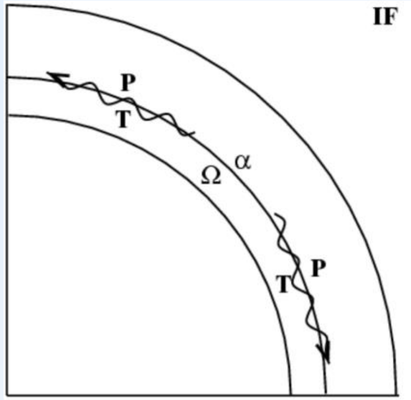
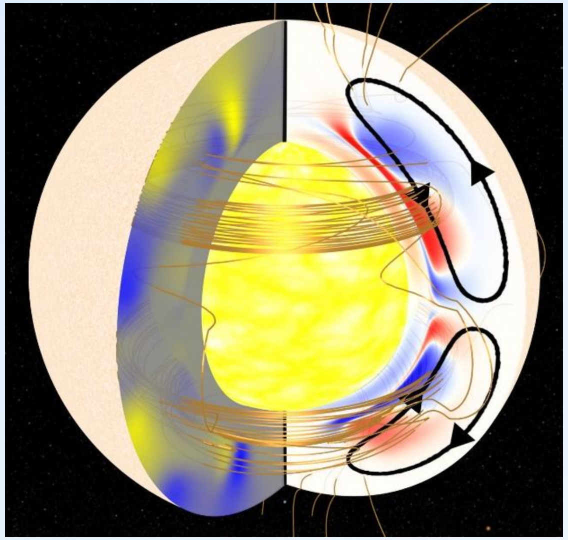
MAX-PLANCK-GESELLSCHAFT

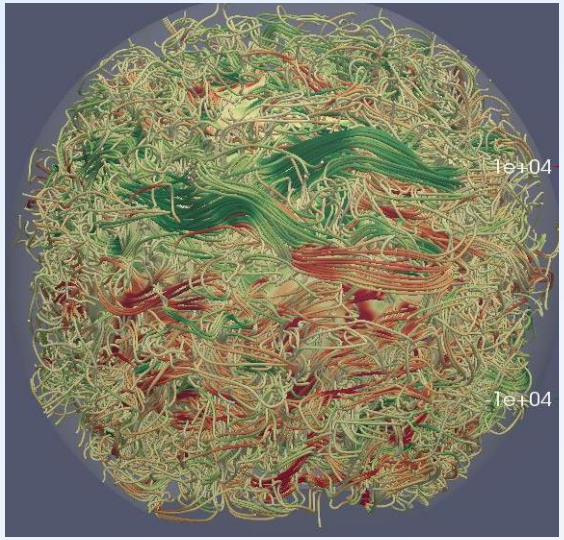












The solar dynamo: changing views

Fan & Fang (2014)

Manfred Schüssler
Max Planck Institute for Solar System Research (MPS)
Göttingen, Germany

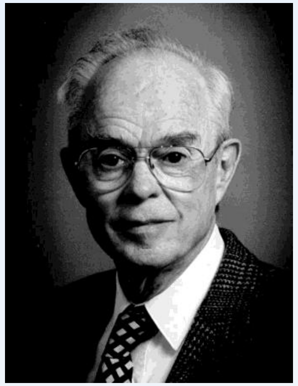
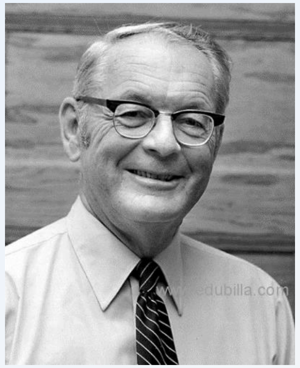
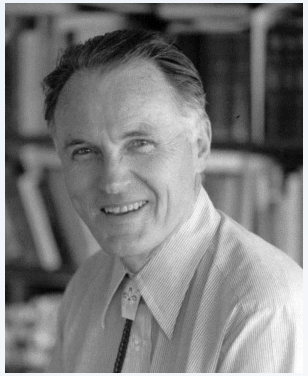
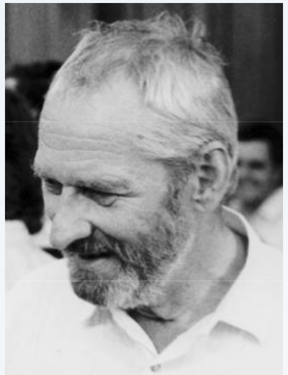
Outline

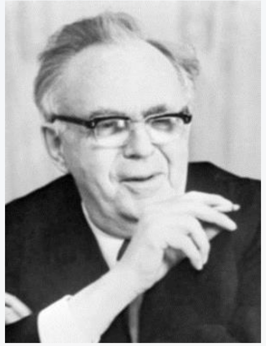
- A brief history
- Challenges to the „current paradigm”
- Babcock-Leighton redux
- Cycle variability

Outline

- **A brief history**
- Challenges to the „current paradigm”
- Babcock-Leighton redux
- Cycle variability





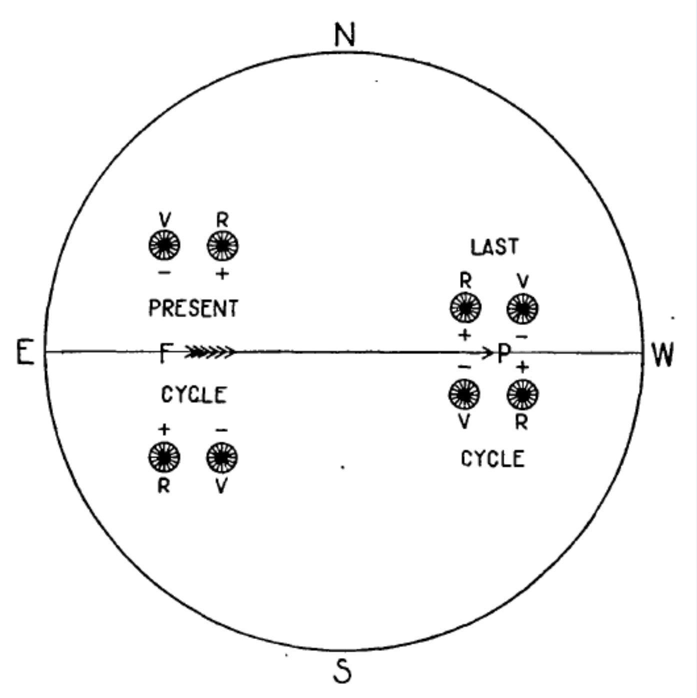
The era of the pioneers I

THE MAGNETIC POLARITY OF SUN-SPOTS[1]

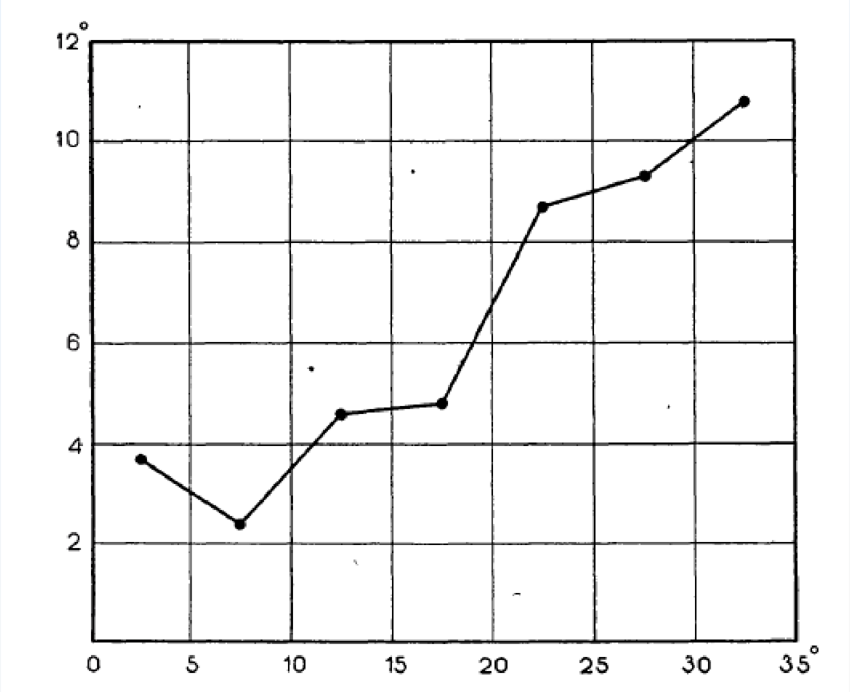
BY GEORGE E. HALE, FERDINAND ELLERMAN, S. B. NICHOLSON, AND A. H. JOY

ApJ 49, 153 (**1919**)

George Ellery Hale



The 22-year magnetic cycle



Tilt angle of sunspot groups as f(λ)

The era of the pioneers II

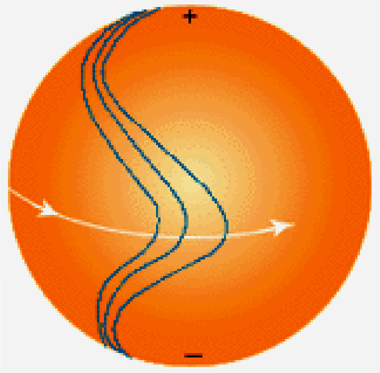
J. Larmor

T. G. Cowling

- Larmor (1919):
  „How could a rotating body such as the sun become a magnet?"

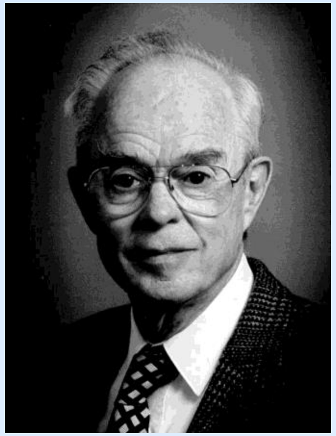
- Cowling (1933):
  „The theory proposed by Sir Joseph Larmor (...) is examined and shown to be faulty"

- Cowling (1951):
  Generation of toroidal from poloidal field
  by differential rotation („Ω-effect")
  But toroidal $\rightarrow$ poloidal ???



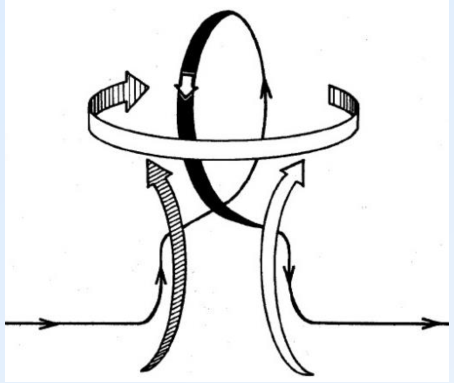
- Babcock & Cowling (1953)
  „ ... one does not expect an irregular cause to build up to give a regular effect."

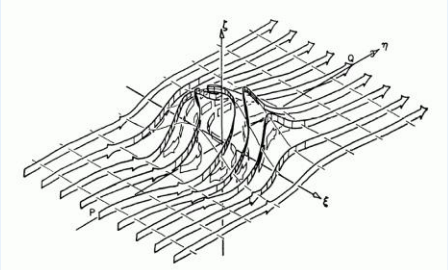
The era of the pioneers III

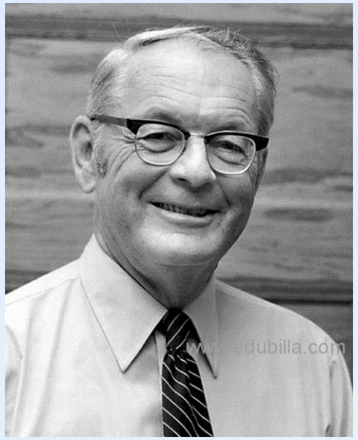


Coriolis force

Eugene N. Parker
(1955)

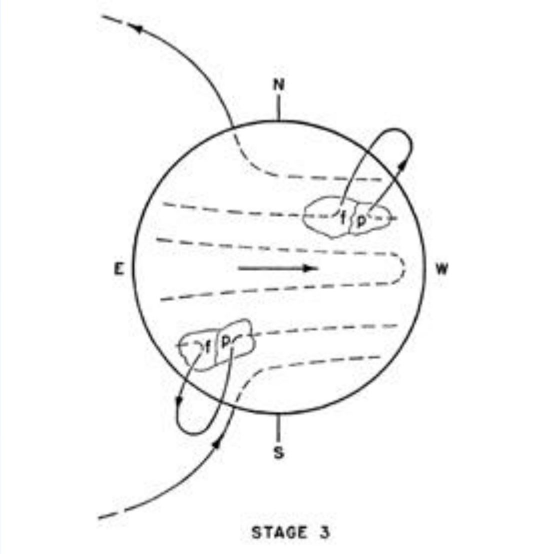


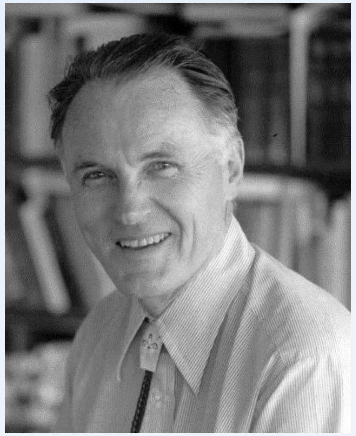




Sunspot group tilt

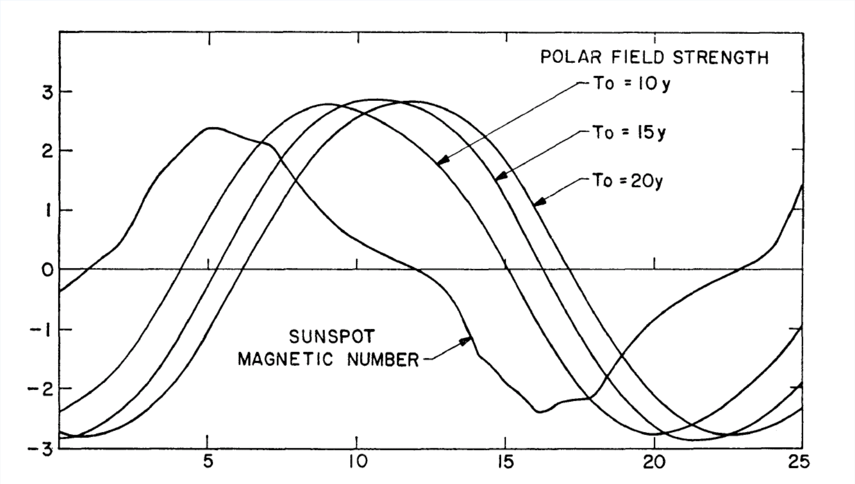
Horace W. Babcock
(1961)



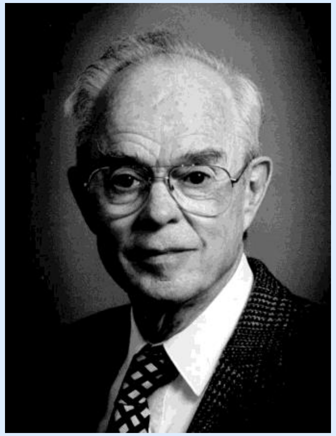


Random walk

Robert B. Leighton
(1964, 1969)

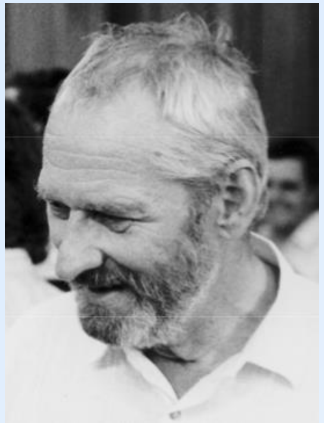


The era of the pioneers IV



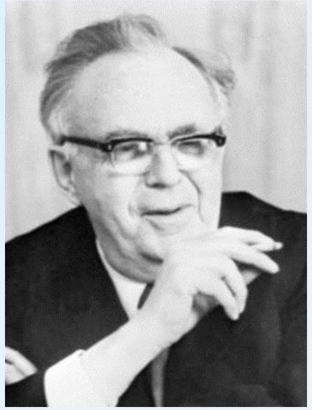
Coriolis force

E. N. Parker
(1955)

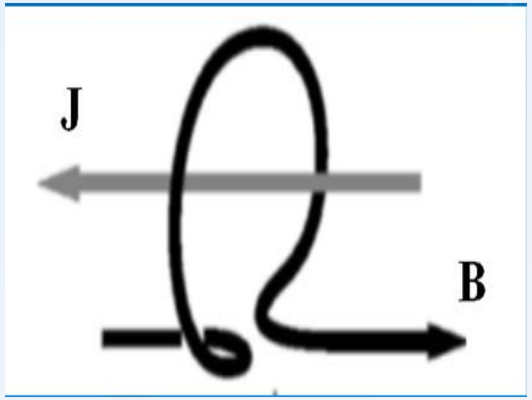


F. Krause

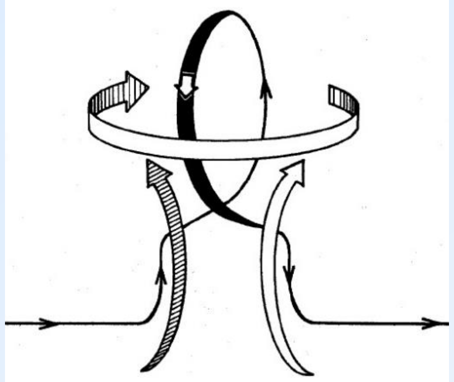
K.-H. Rädler

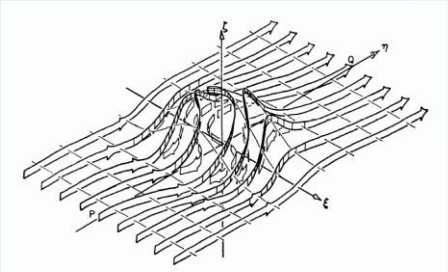


M. Steenbeck



The turbulent dynamo





$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times \left(\langle \mathbf{u} \rangle \times \langle \mathbf{B} \rangle + \alpha \langle \mathbf{B} \rangle \right) - \nabla \times \left[(\eta + \beta) \nabla \times \langle \mathbf{B} \rangle \right]$$

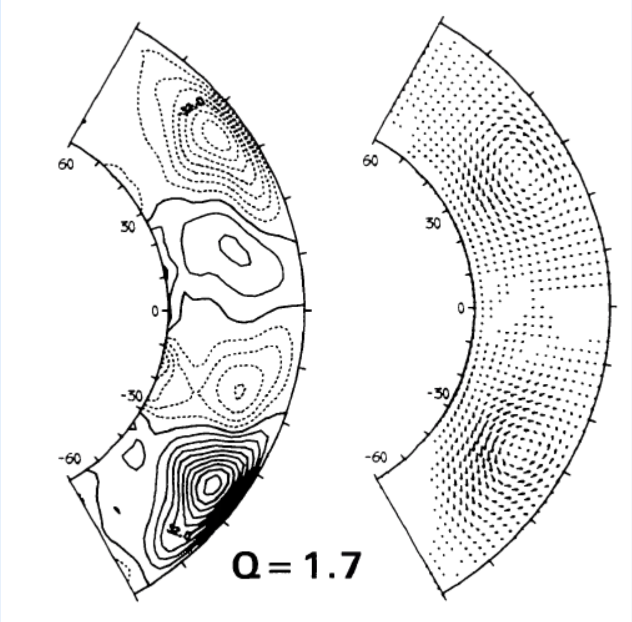
1970s - the glorious decade:

- α-effect dynamo „industry”:
  models for the Sun, stars, planets, galaxies, accretion disks,...
- nonlinear effects: „cut-off-α”, Malkus-Proctor effect, time delays,...

Challenges, blows & new horizons

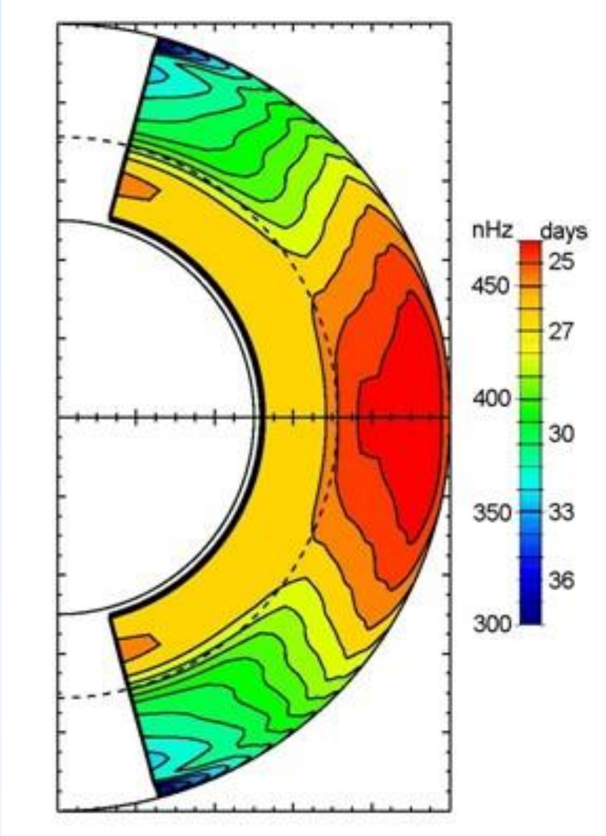
Magnetic buoyancy
(Parker, 1975):
$\rightarrow$ Magnetic flux lost from the convection zone within months?

First 3D-MHD simulations
(Gilman & Miller, 1981; Gilman, 1983):
$\rightarrow$ No solar-like cycles?



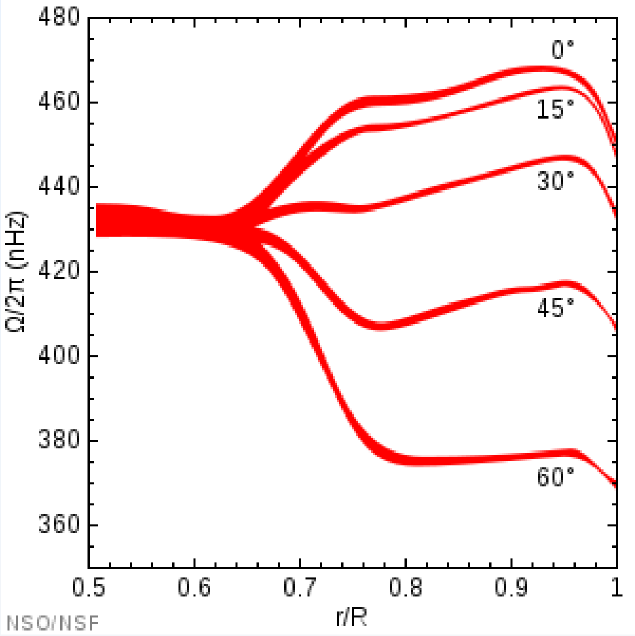
(Gilman, 1993)

Helioseismology
Prediction of mean-field dynamo models: $d\Omega/dr < 0 \rightarrow$ refuted



Contours of equal rotation period
(Howe et al., 2005)

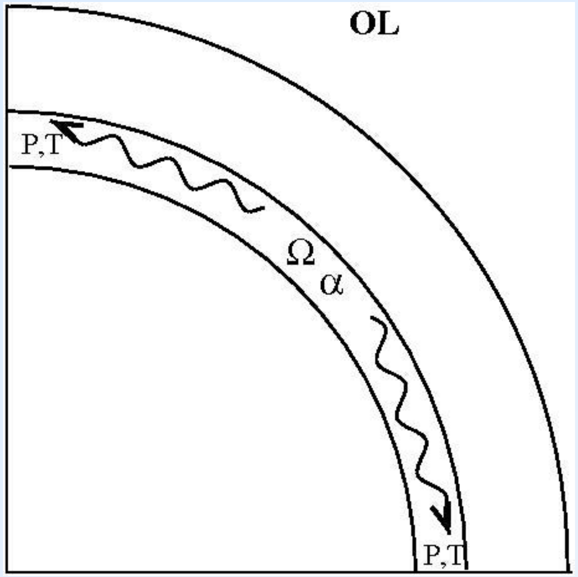
Tachocline
(Brown et al., 1989)



Convective overshoot layer
stably stratified at the bottom of the convection zone
(Galloway & Weiss, 1981)

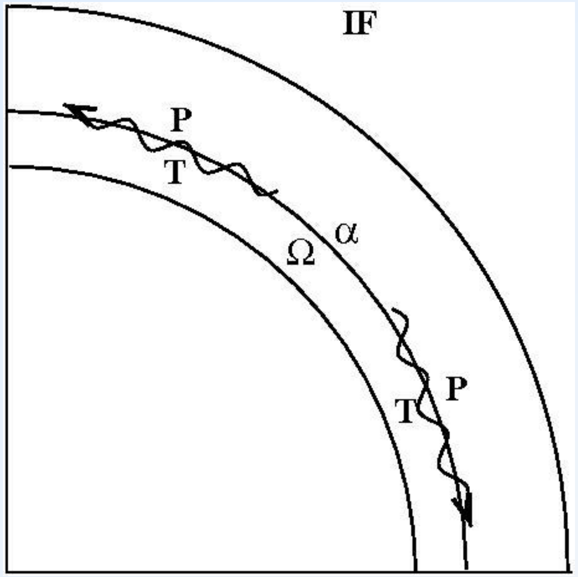
Modified concepts & new challenges

Overshoot-layer dynamo

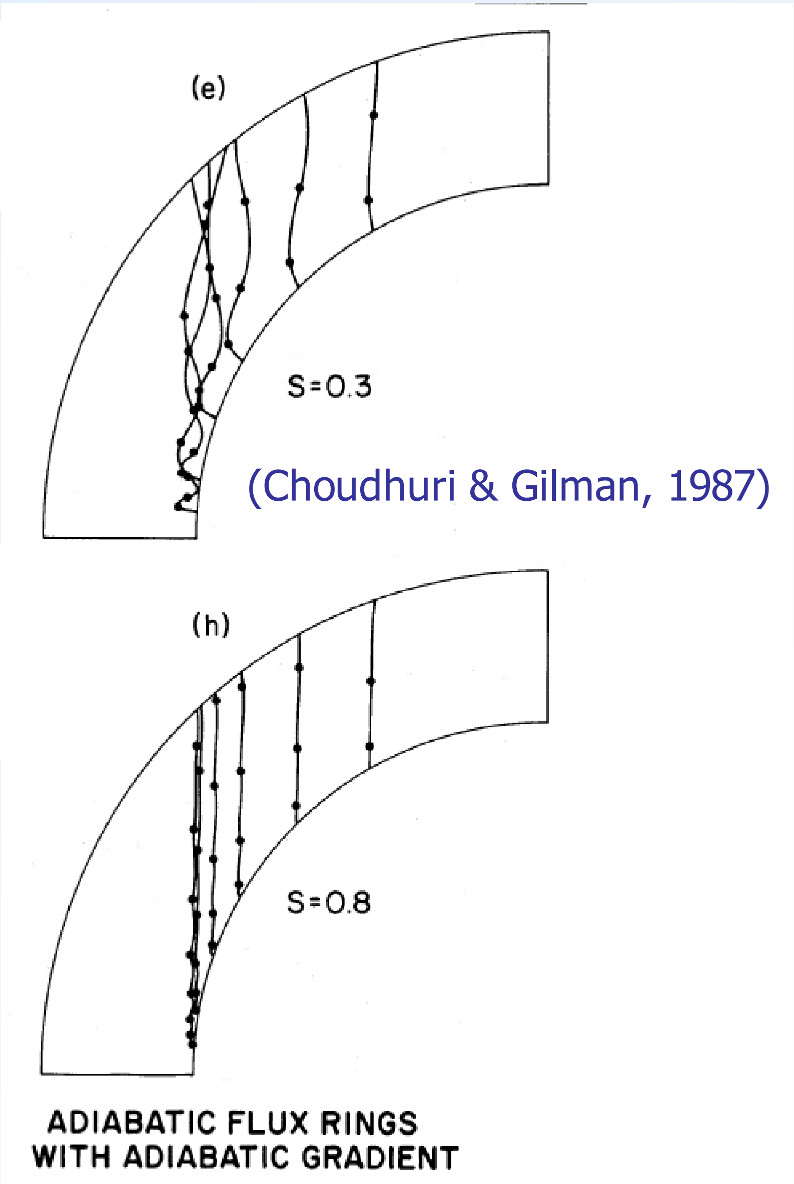


(Galloway & Weiss, 1983)

Interface dynamo



(Parker, 1993)

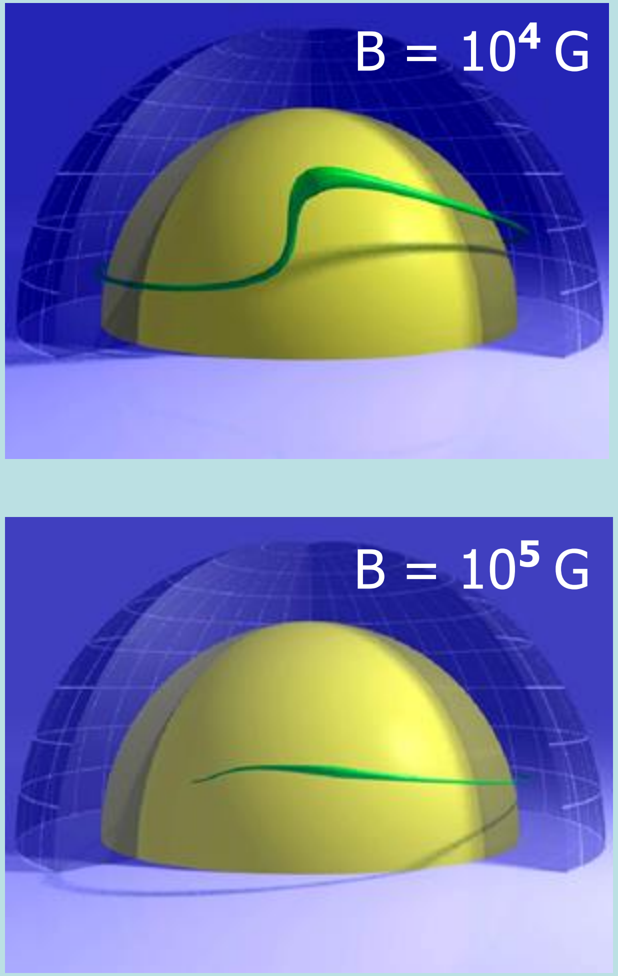


(Choudhuri & Gilman, 1987)

Buoyantly rising flux tubes

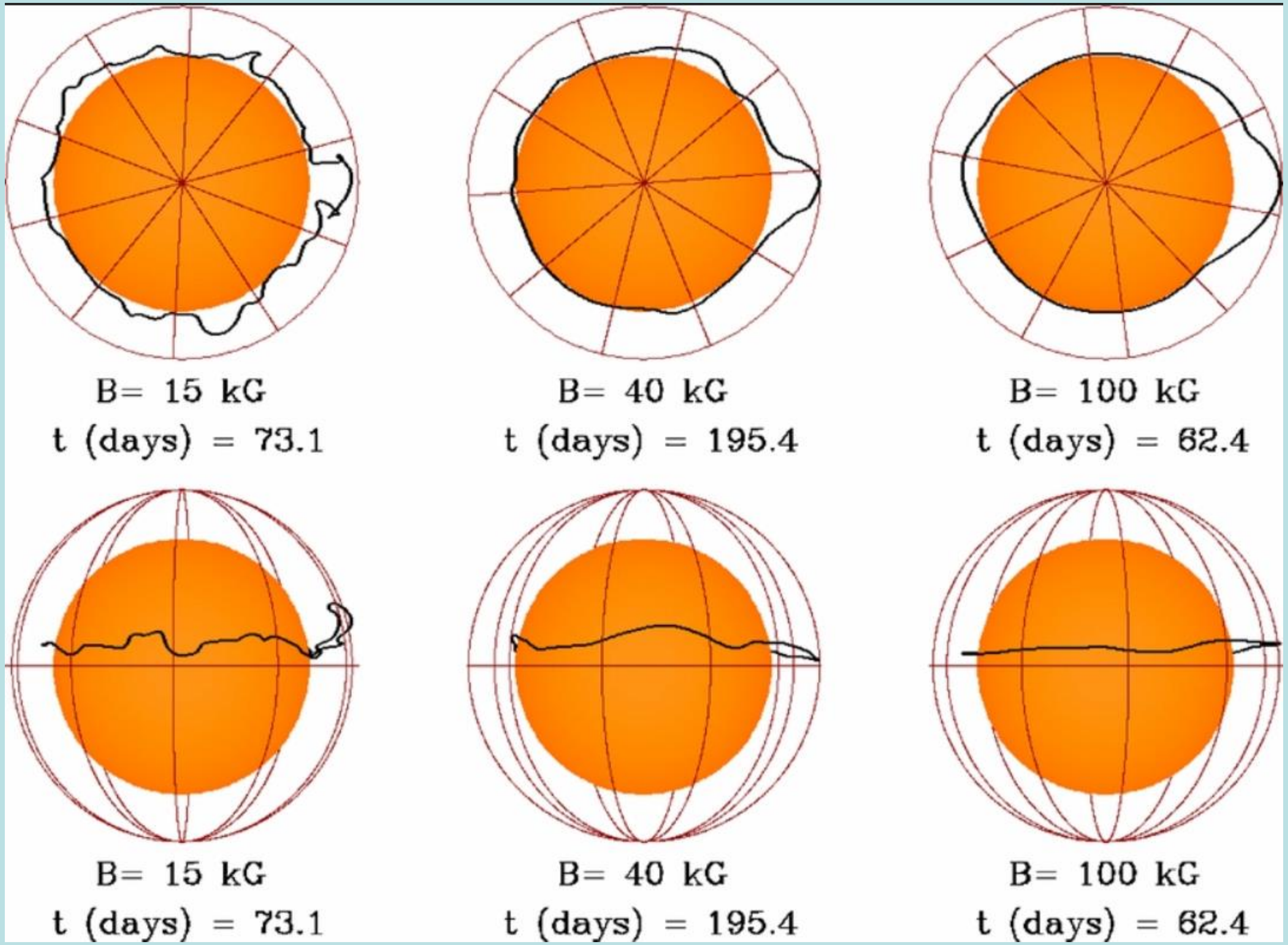
Modified concepts & new challenges

Buoyantly rising thin flux tubes…



(Caligari et al., 1995)

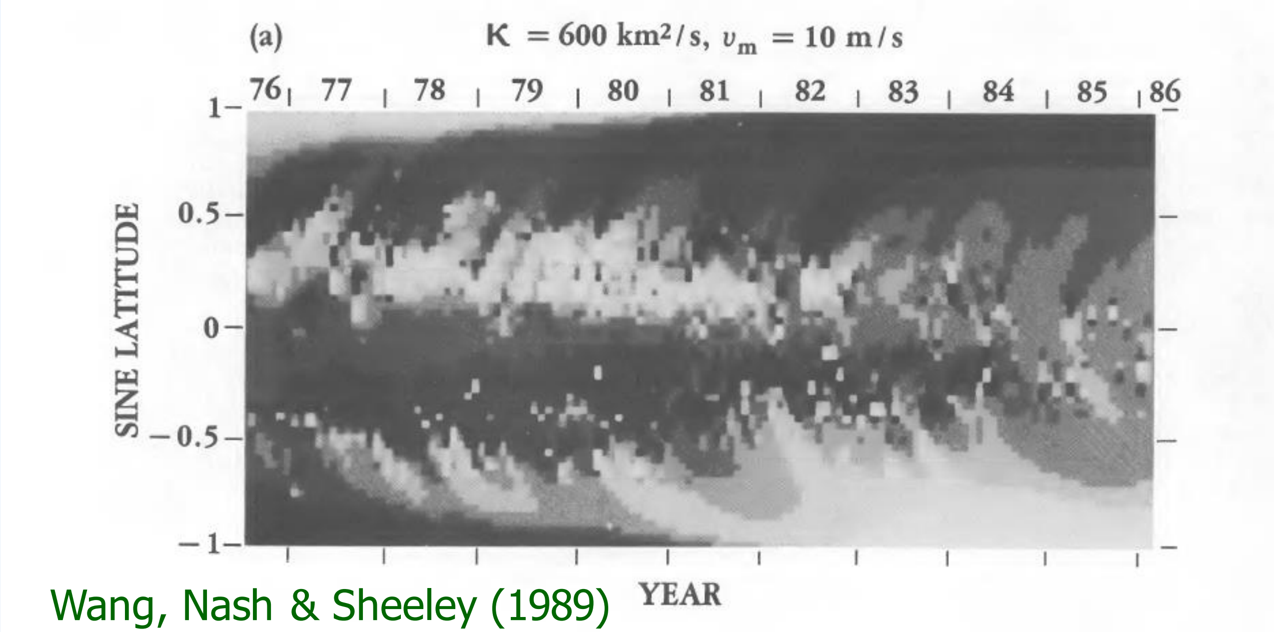
… in simulated 3D convection



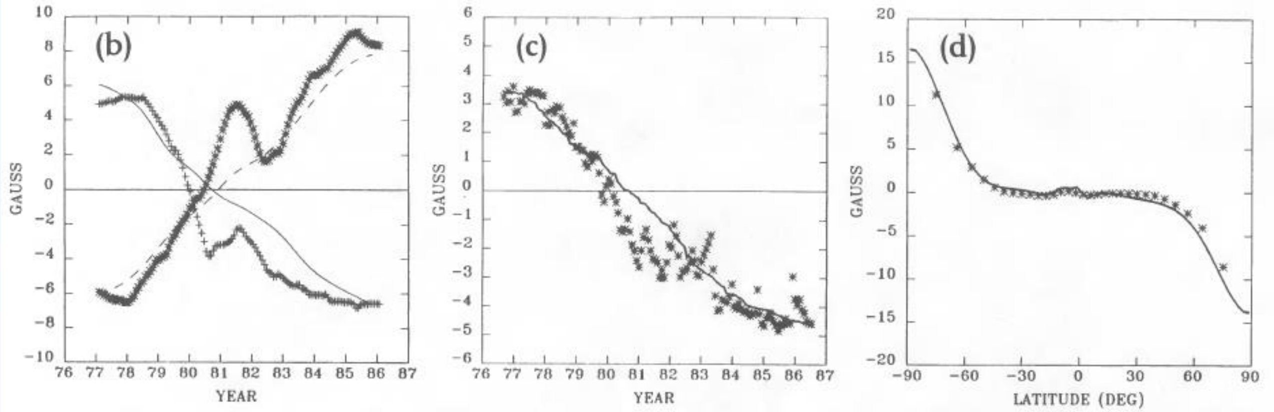
(Weber et al., 2011)

A new twist…

Surface flux transport simulations



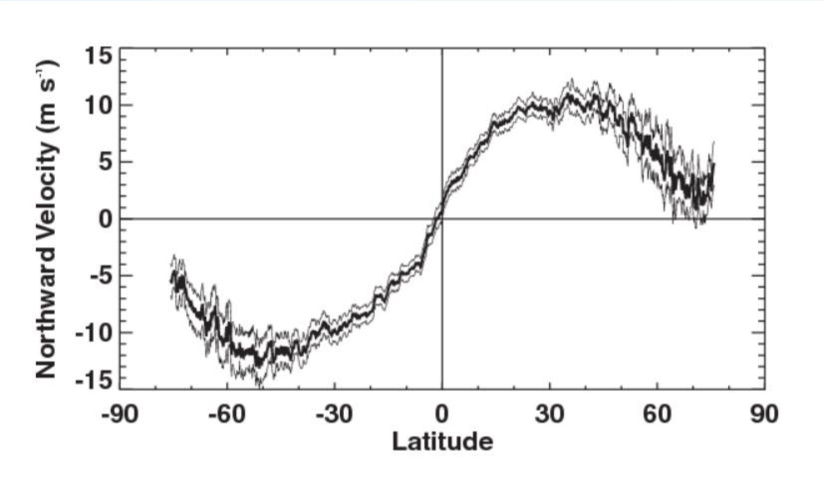
Wang, Nash & Sheeley (1989)



Polar field Axial dipole Surface field as f(λ)

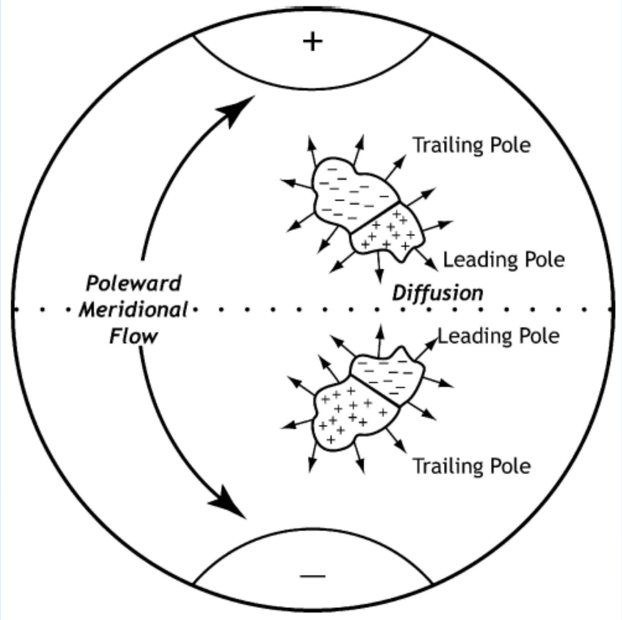
Poleward meridional surface flow

Duvall (1979, Howard & LaBonte (1981), Andersen (1987) …

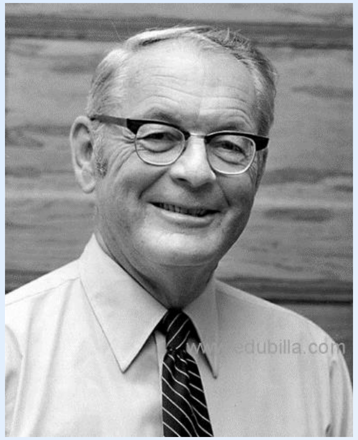


Hathaway & Rightmire (2010)

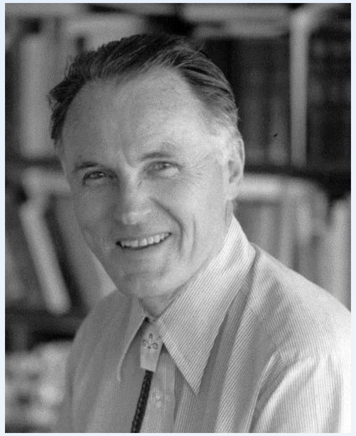
The pioneers (rediscovered)



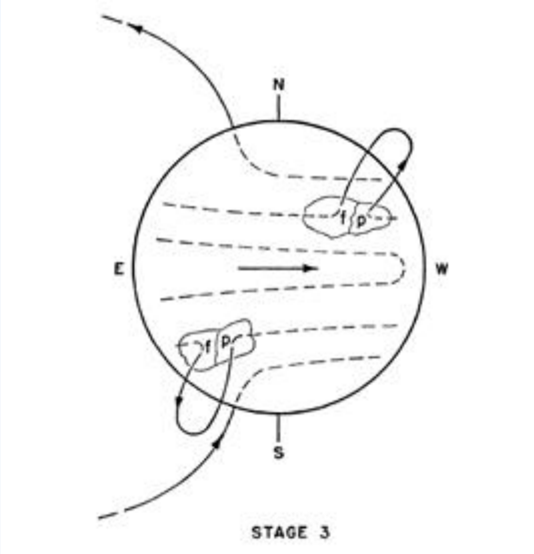
Wang (2005)



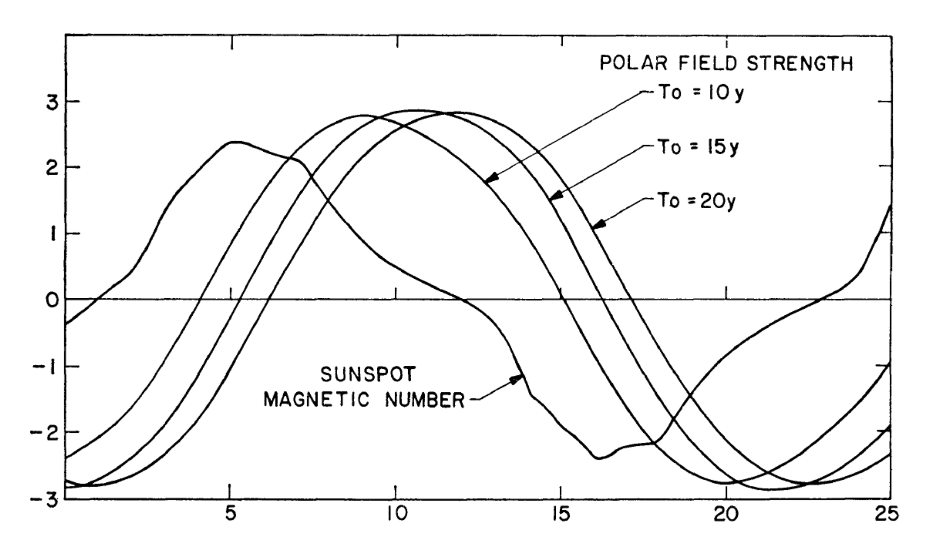
Horace W. Babcock



Robert B. Leighton

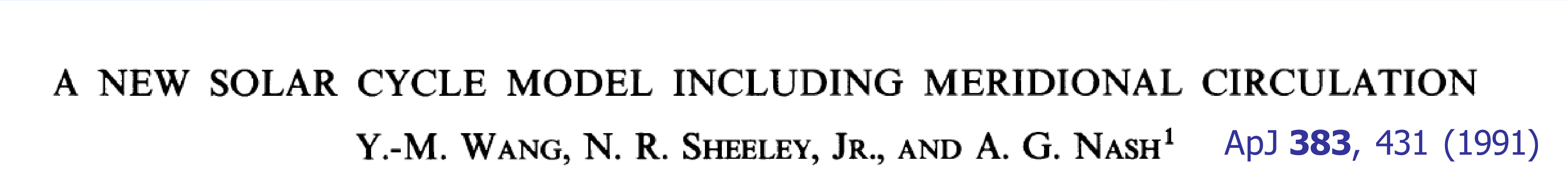


1961

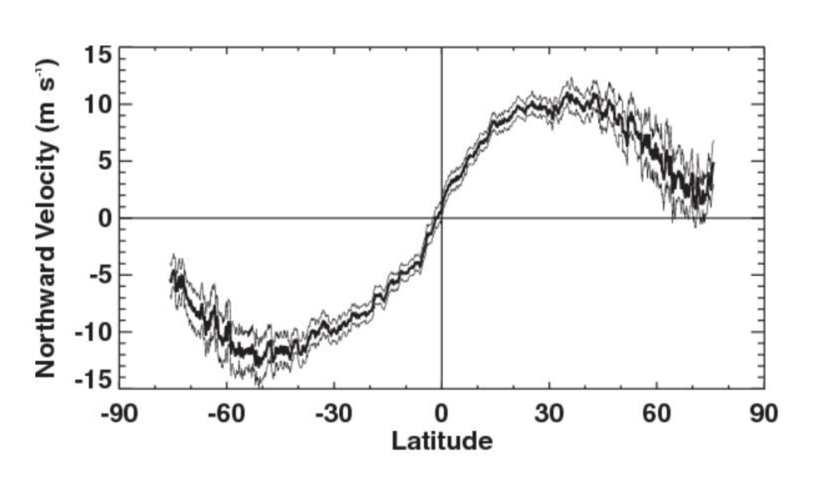


1964

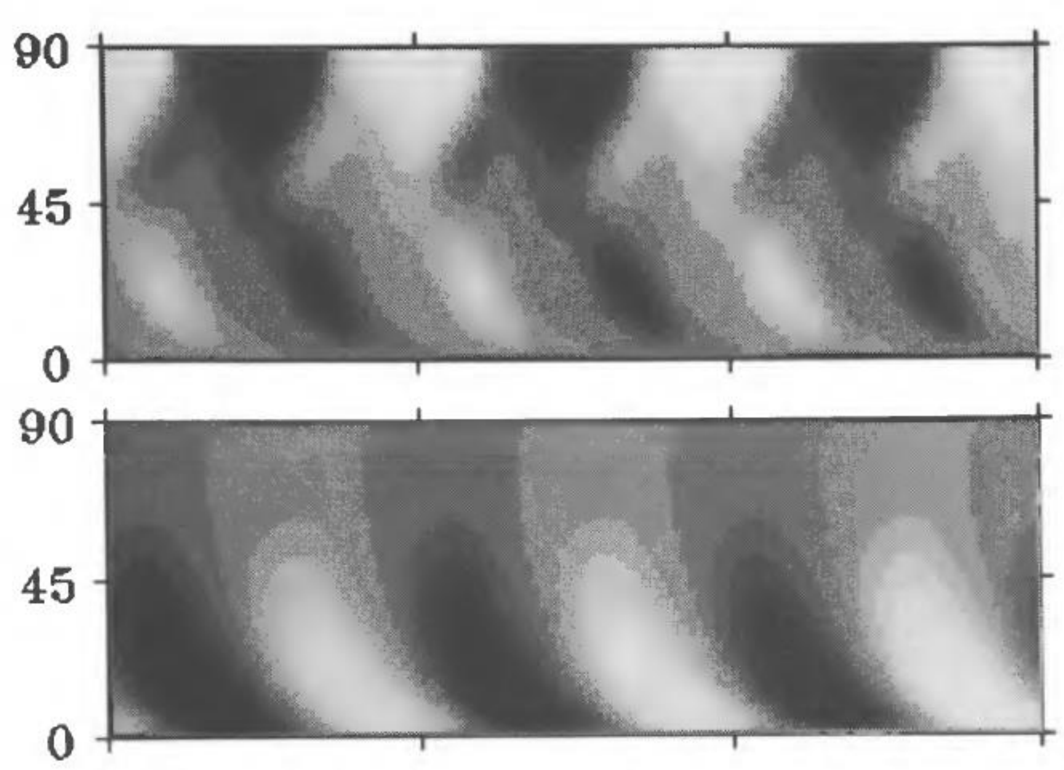
Babcock-Leighton rediscovered


A NEW SOLAR CYCLE MODEL INCLUDING MERIDIONAL CIRCULATION

Y.-M. WANG, N. R. SHEELEY, JR., AND A. G. NASH[1] ApJ **383**, 431 (1991)



Hathaway & Rightmire (2010)



surface $\mathbf{B}_r$

subsurface $\mathbf{B}_\phi$

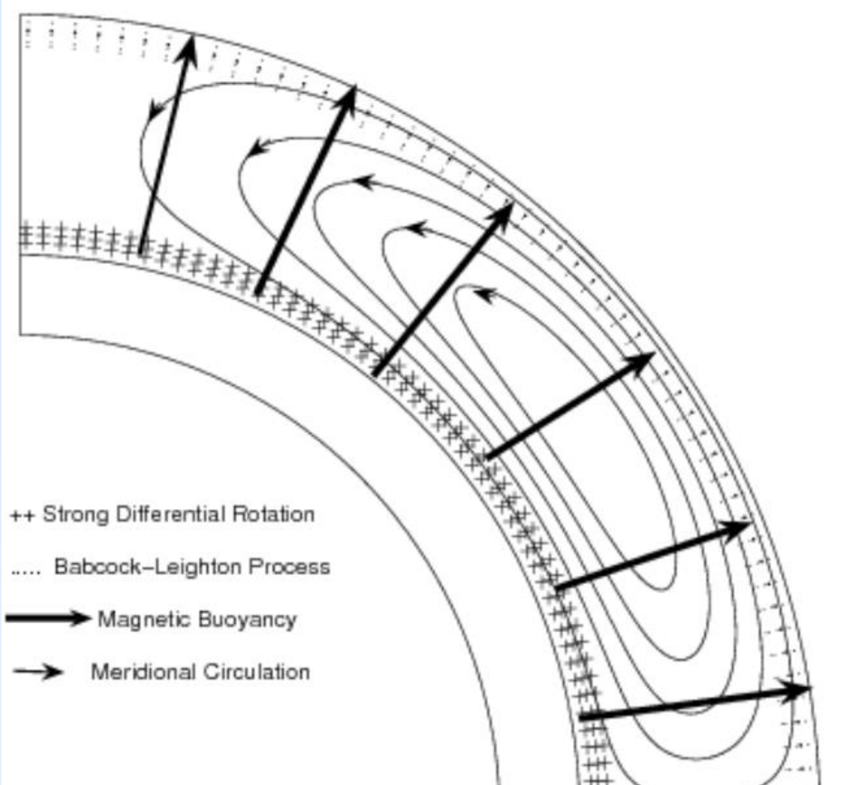
Time-latitude diagrams

Does the subsurface return flow lead to the equatorward propagation of the activity belts?

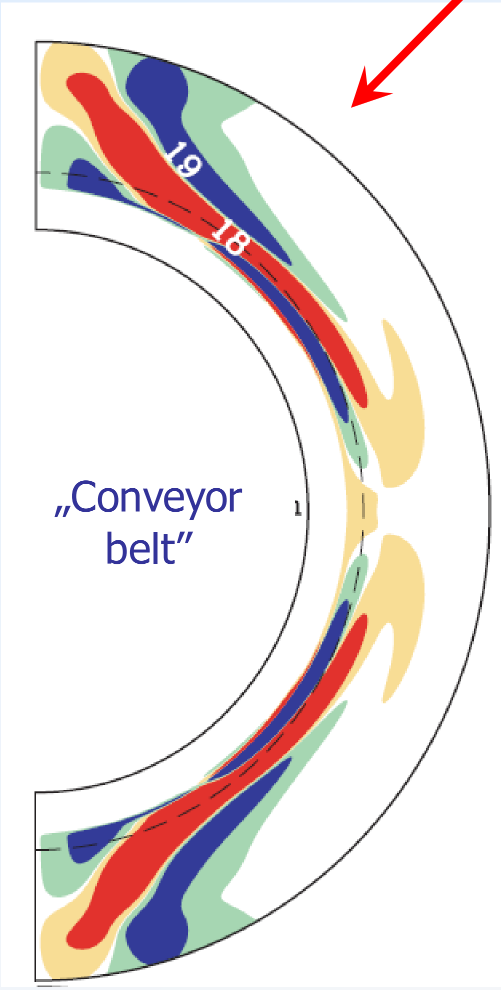
Flux transport dynamos

The current paradigm…

„Dynamo wars": advection-dominated vs. diffusion-dominated

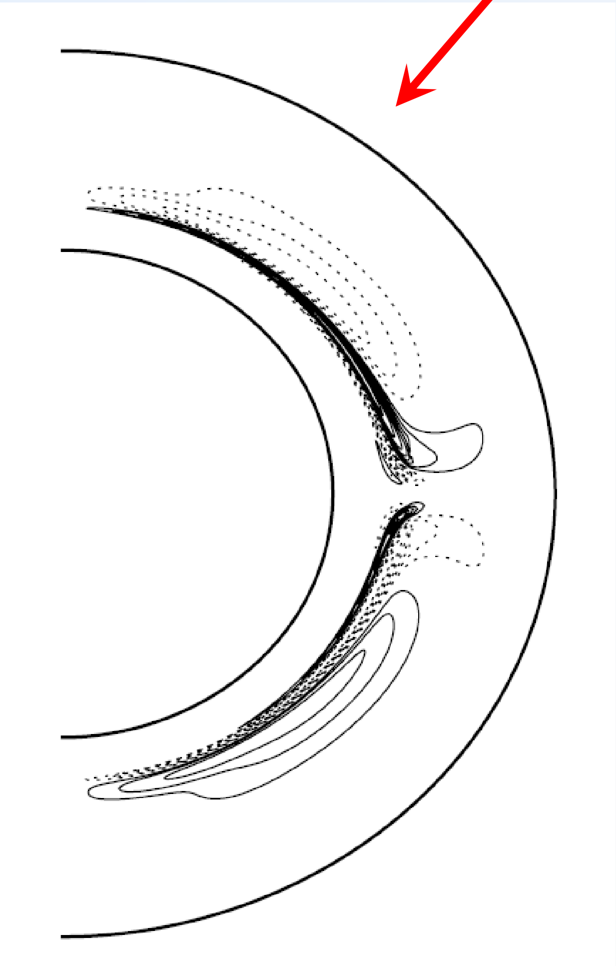


(Karak et al., 2014)



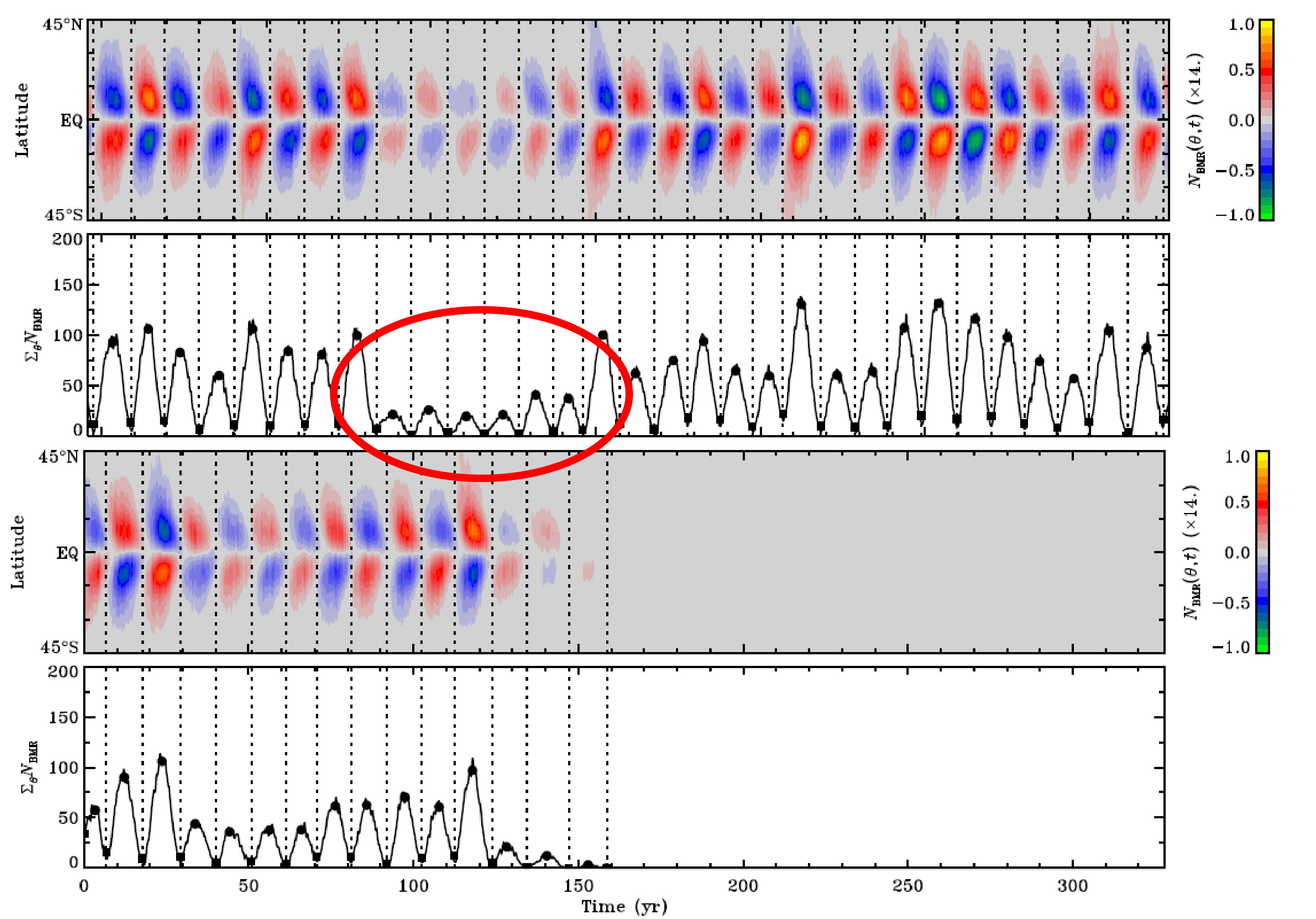
„Conveyor belt"

(Dikpati & Gilman, 2006)

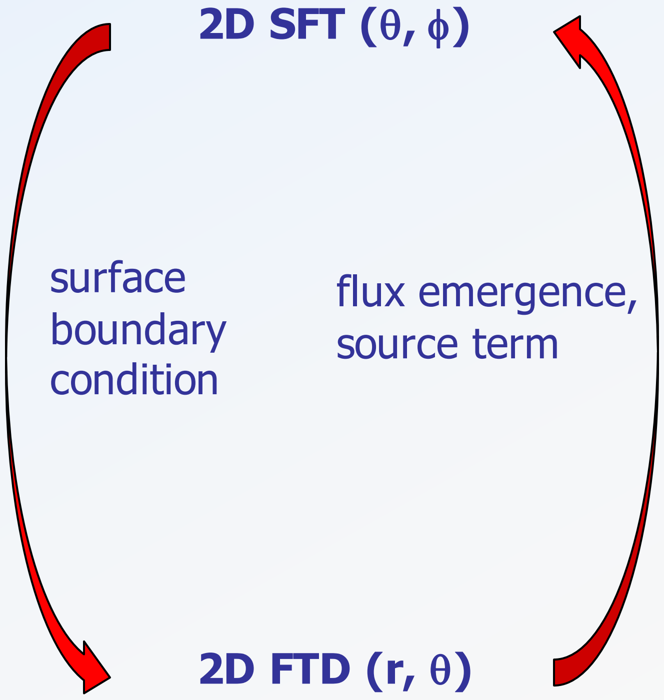


(Jiang et al., 2007)

Flux transport dynamos



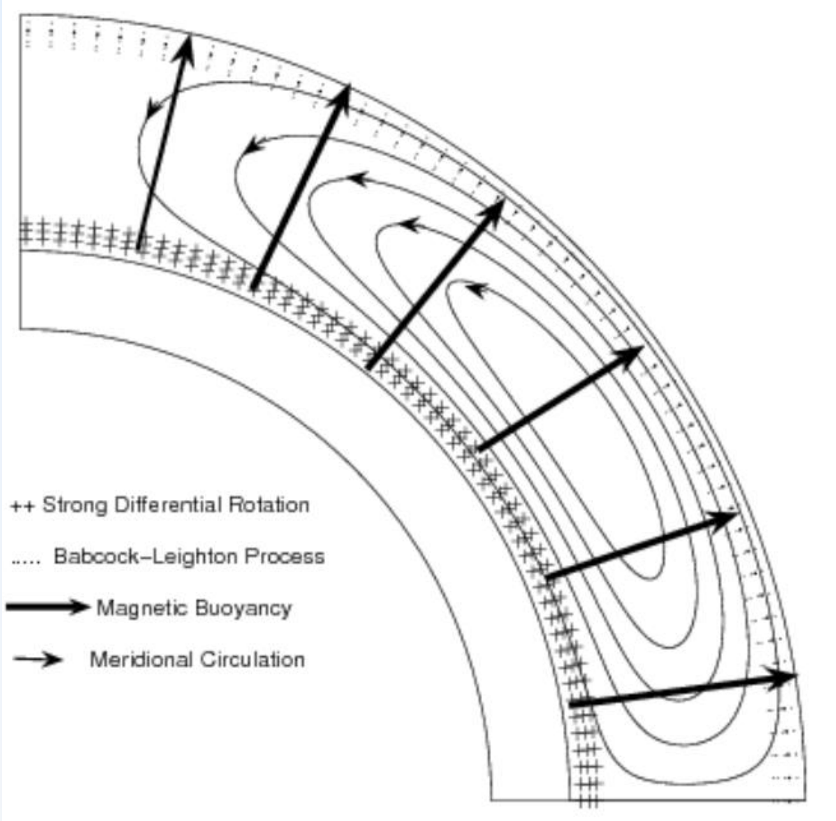
Lemerle et al. (2015, 2017)
„2D × 2D model"



Model parameters fixed by observational constraints via a genetic algorithm.

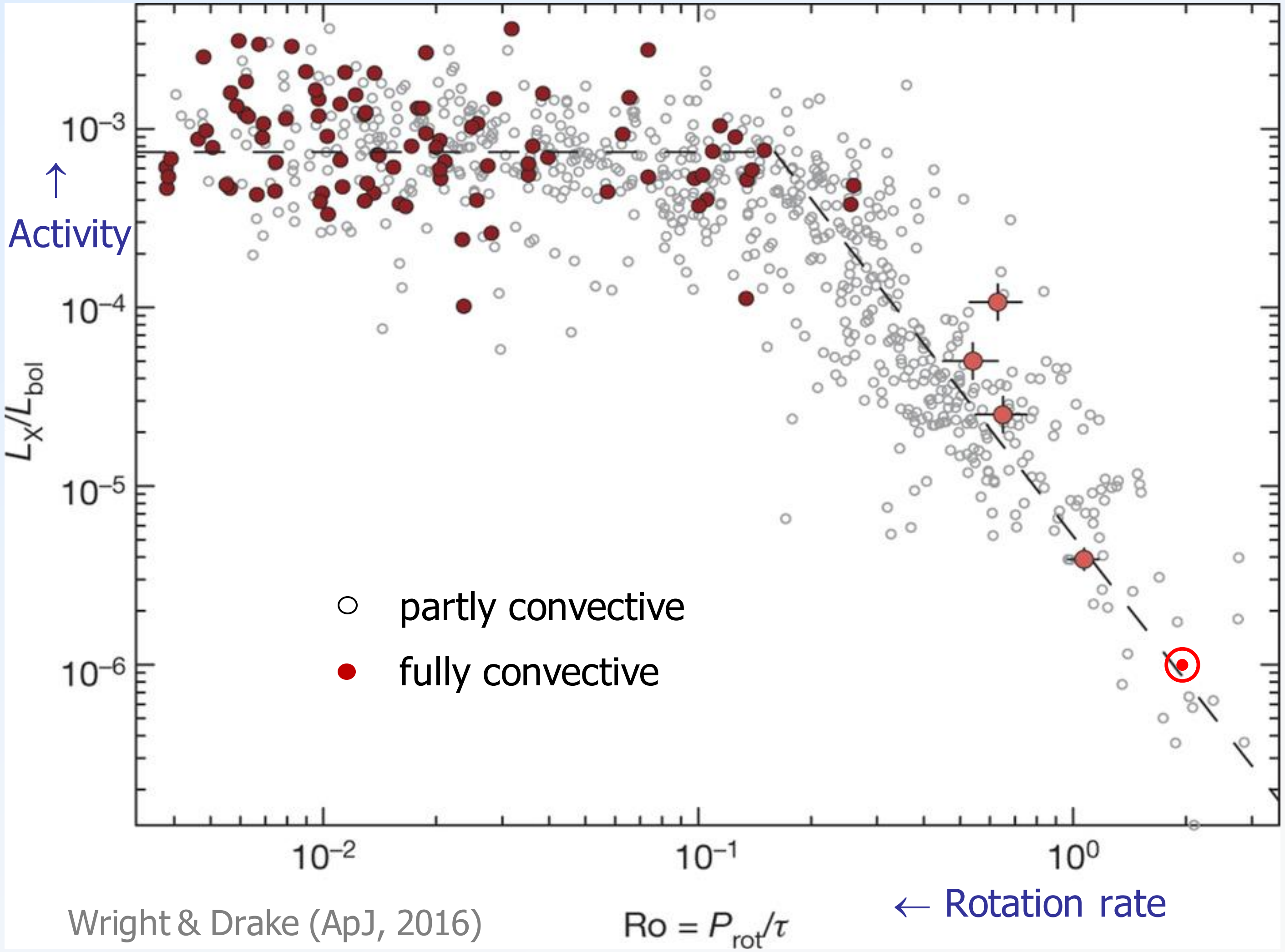
Outline

- ➢ A brief history
- ➢ **Challenges to the „current paradigm"**
- ➢ Babcock-Leighton redux
- ➢ Cycle variability



Tachocline: is it relevant for the dynamo?

- Absence of a significant cycle variability of tachocline rotation
  ($E_{kin} \approx E_{mag}$ for B ~ 10^5 G; Rempel, 2006)
- Maintainance of a magnetic tachocline?
  (Spruit, 2010)
- Toroidal flux generated by latitudinal differential rotation from flux of the polar field sufficient to supply the emerged flux
  (Cameron & S., 2015)
- Partly and fully convective stars follow the same activity-rotation law
  (Wright & Drake, 2016)
- Activity cycles shown by ultracool, fully convective dwarfs (≥ M7)
  (Route, 2016)

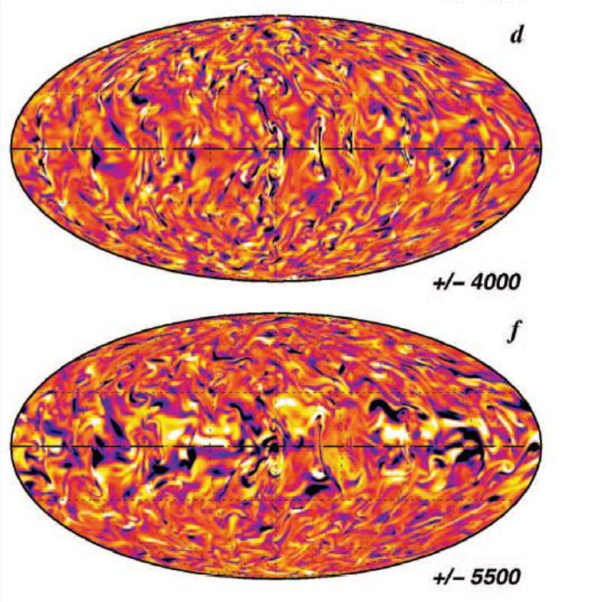


Wright & Drake (ApJ, 2016)

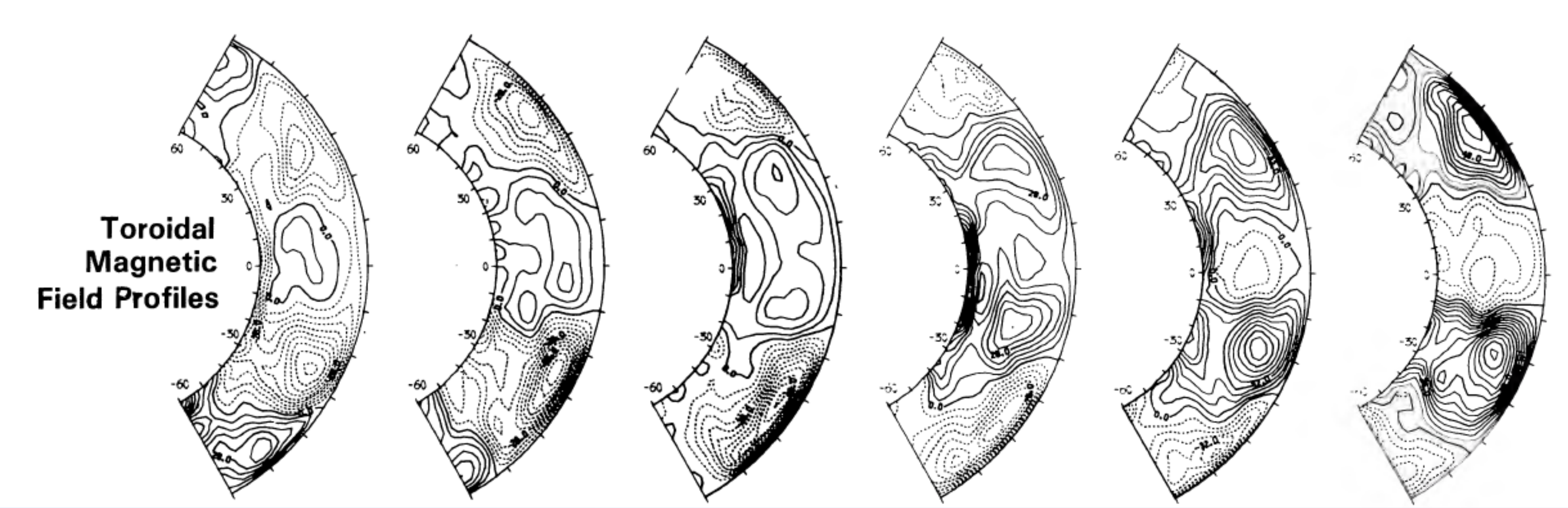
3D MHD simulations

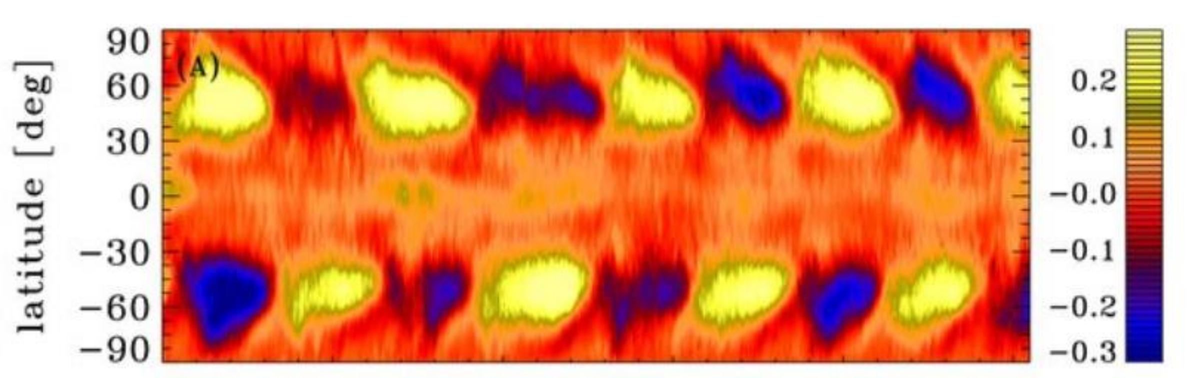
→ Review by Brun & Browning (Liv. Rev. Sol. Phys., 2017)

Brun et al. (2004)

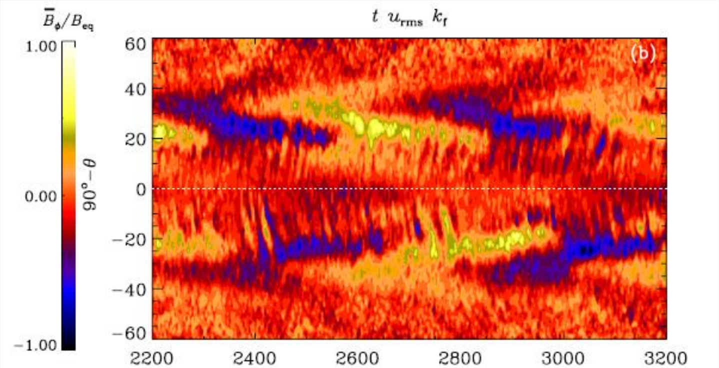


Gilman (1983)





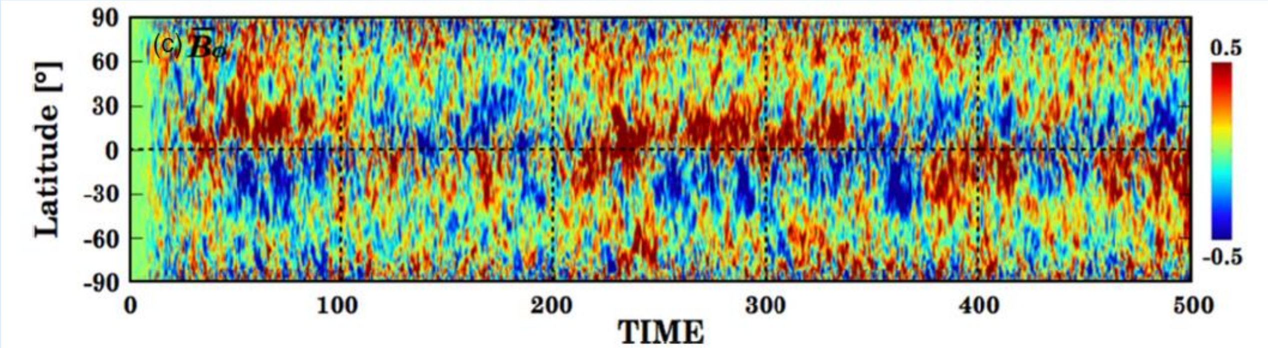
Ghizaru et al. (2011)

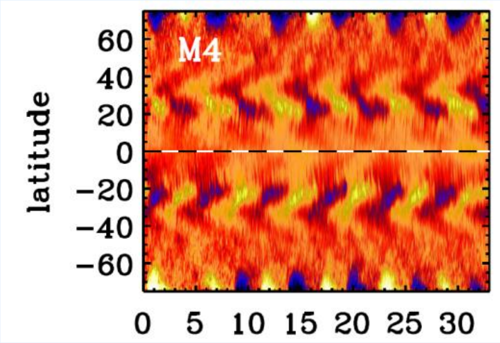


Käpylä et al. (2012)

3D MHD simulations

Masada et al. (2013)

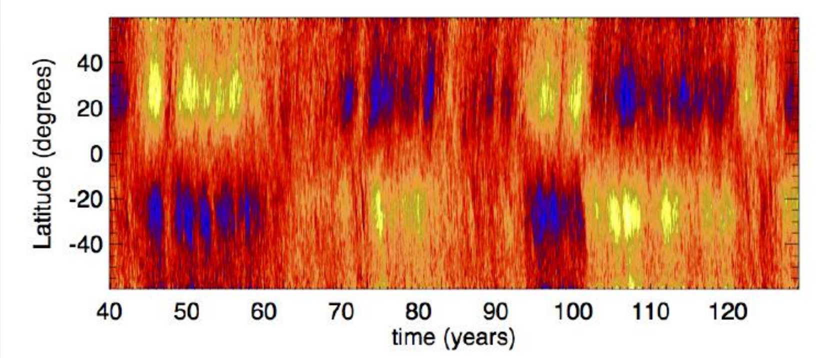


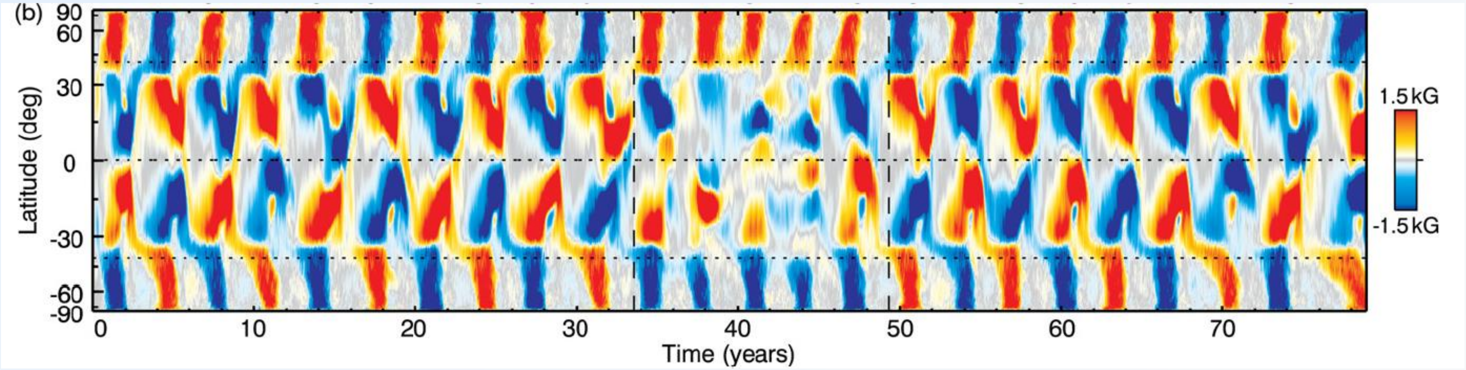


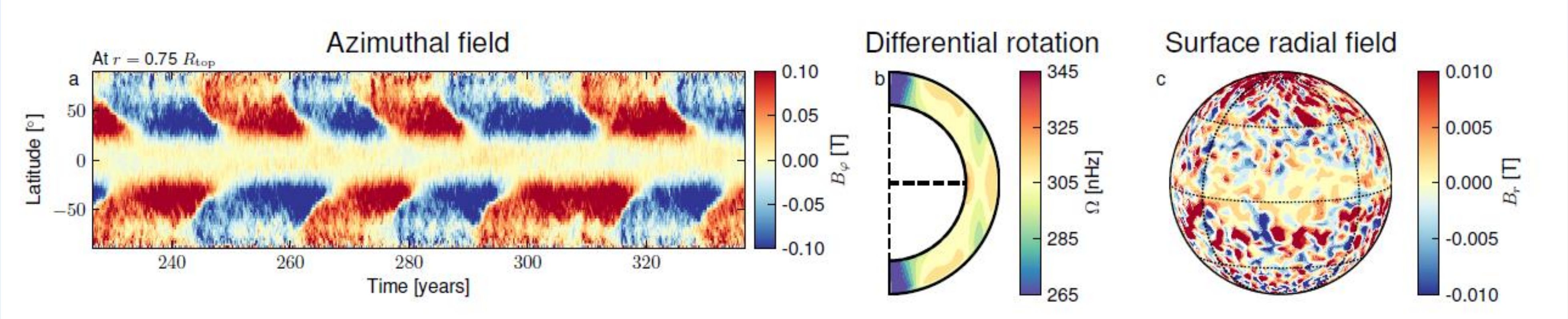
Warnecke (2018)

Augustson et al. (2018)

Fan & Fang (2016)

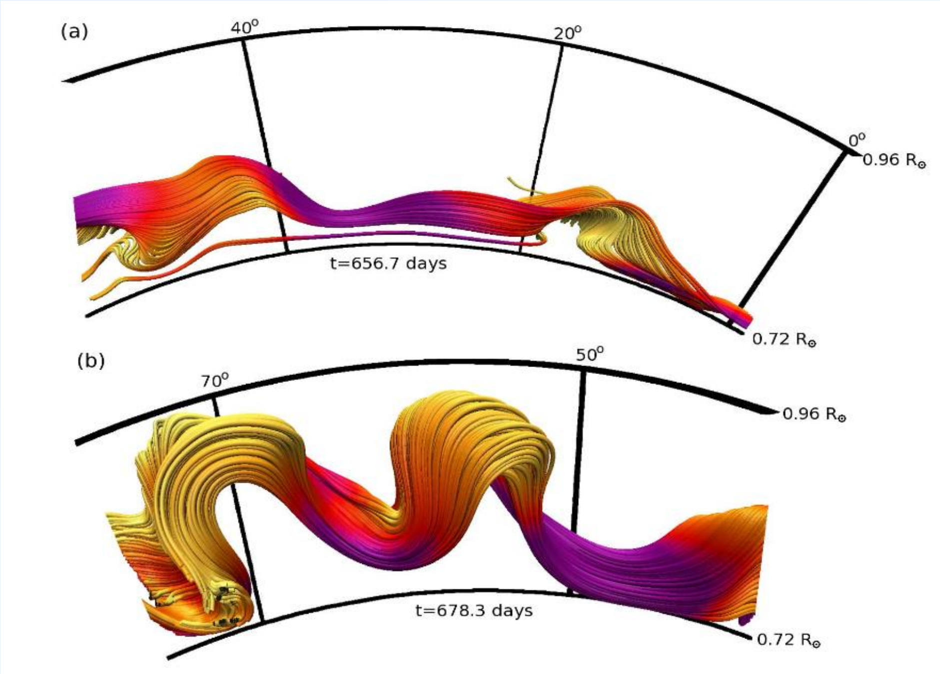




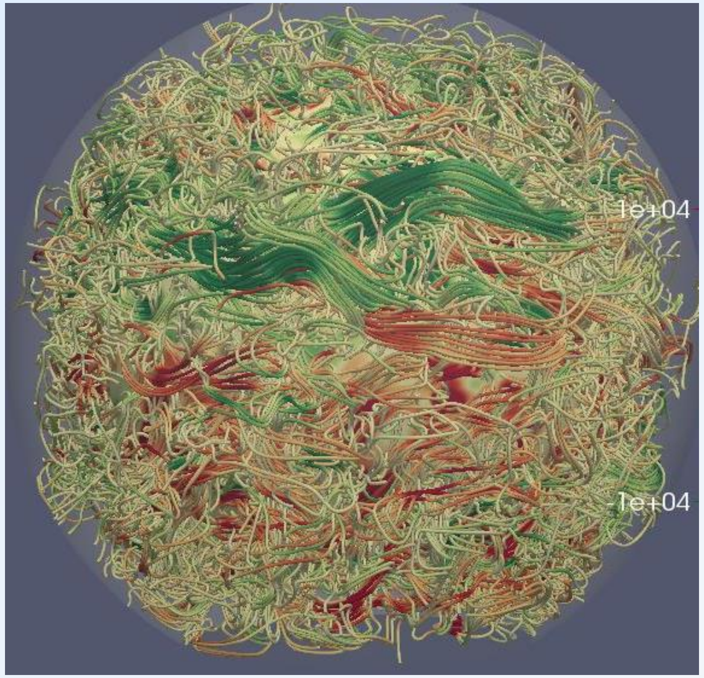


Strugarek et al. (2018)

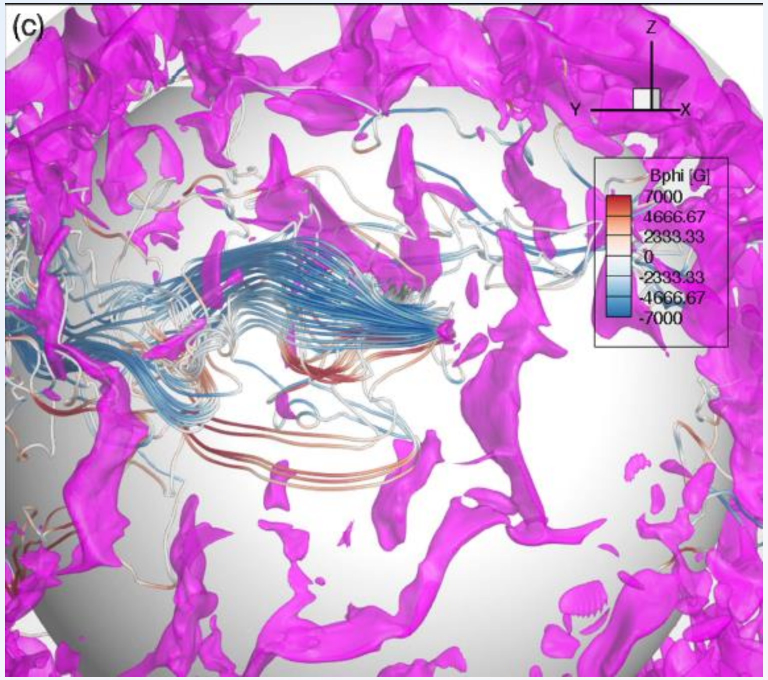
Super-equipartition rising flux loops



Nelson & Miesch (2014)

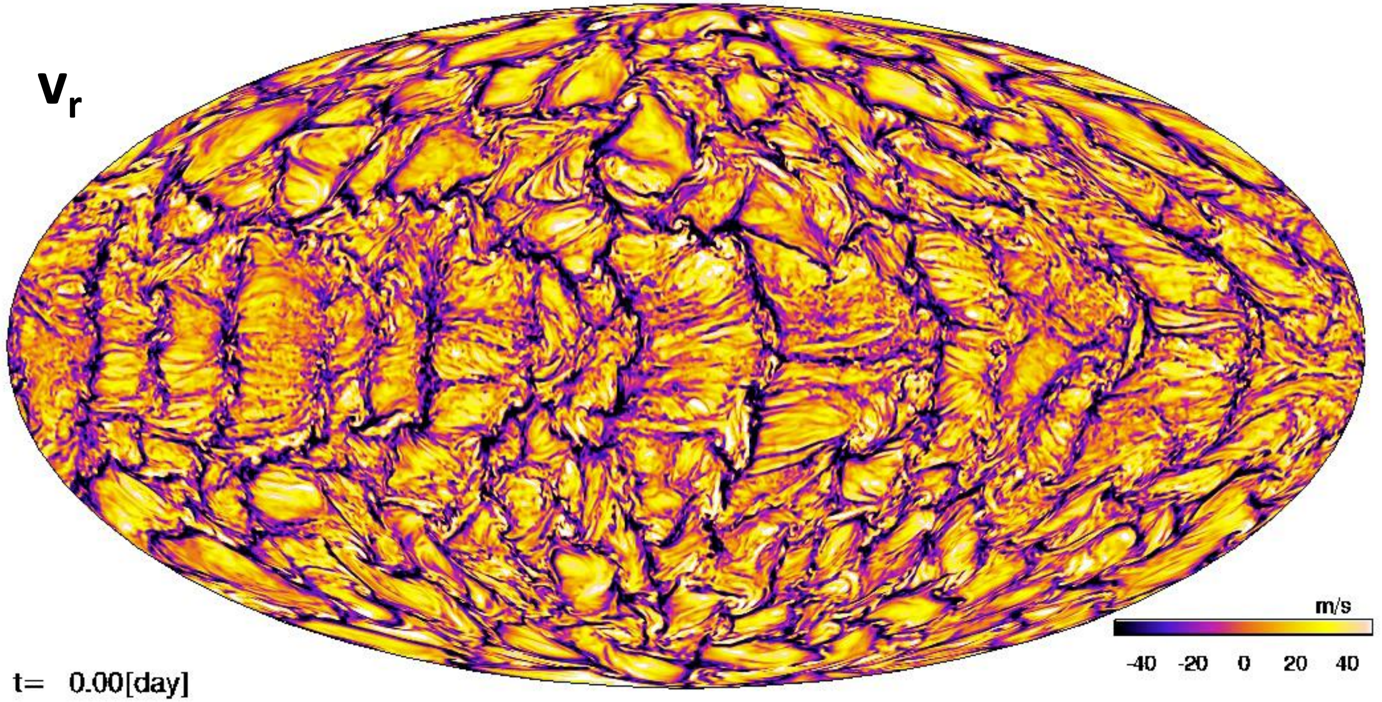


Fang & Fan (2014)

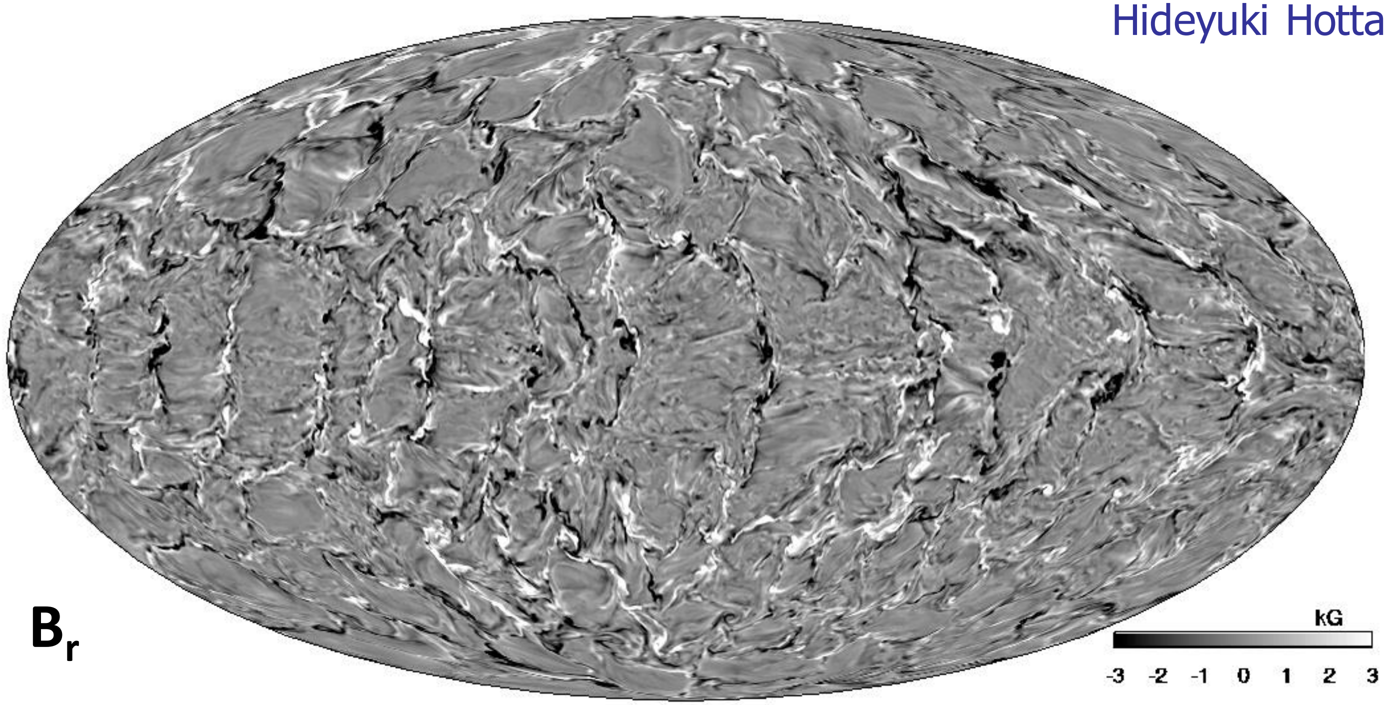


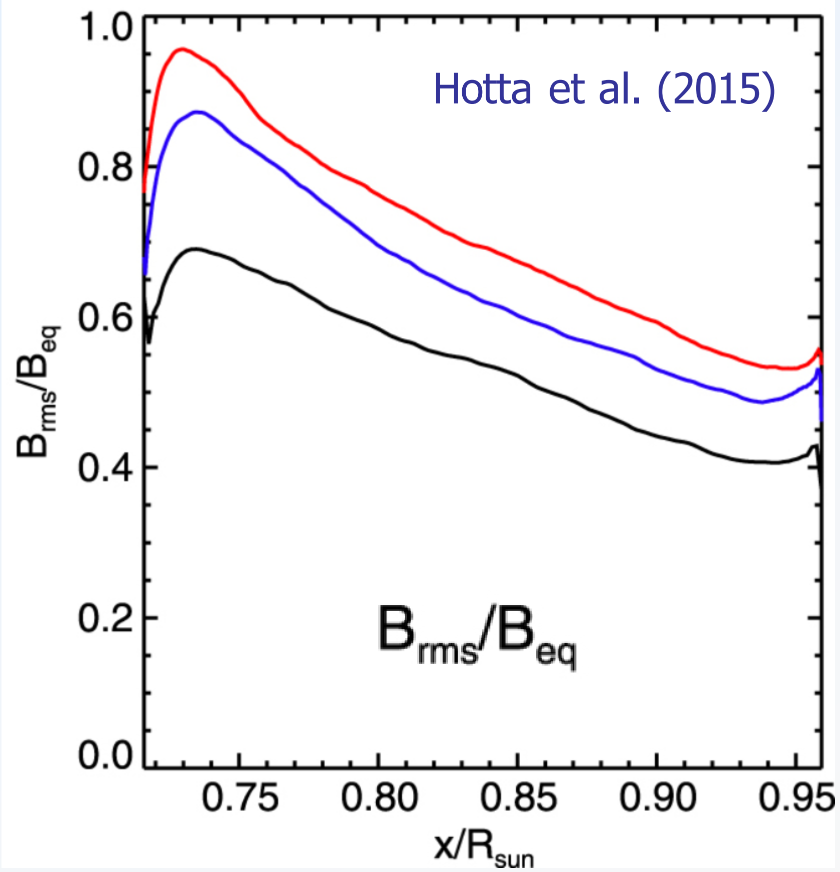
Chen et al. (2017)

Small-scale dynamo action in a convection zone simulation



Hideyuki Hotta



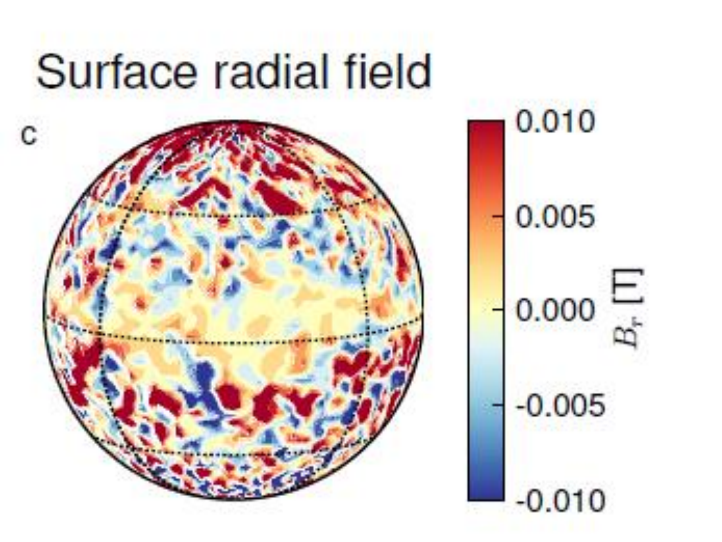


Hotta (2018):
Significant effect on convection, meridional flow, differential rotation…

3D MHD simulations: lessons & limitations

What have we learnt so far (in my view…)?

- self-consistent solar-similar cyclic large-scale dynamo action
  (possible without a tachocline, overshoots layer,…)
- formation of super-equipartition flux concentrations within the convection zone
- importance of small-scale dynamo action
- …more (see following talks)



Strugarek et al. (2018)

Which are the limitations of currently feasible 3D MHD simulations?

- convergence as the resolution is increased?
- too much power in large-scale flows $\rightarrow$ too strong α-effect?
- solar-like latitudinal differential rotation not reproduced under „solar conditions"
- no proper reproduction of flux emergence
  (buoyancy of thin flux concentrations maintained?)

Problem: reality checks?

Outline

- A brief history
- Challenges to the „current paradigm”
- **Babcock-Leighton redux**
- Cycle variability

Is the BL model still relevant?

Ockham chooses a razor

- **Convection zone: largely „terra incognita"**

  FTD models require extensive parametrization
  and 3D MHD models probably run in the wrong physical regime
  $\rightarrow$ a fully realistic dynamo model is not available at the moment

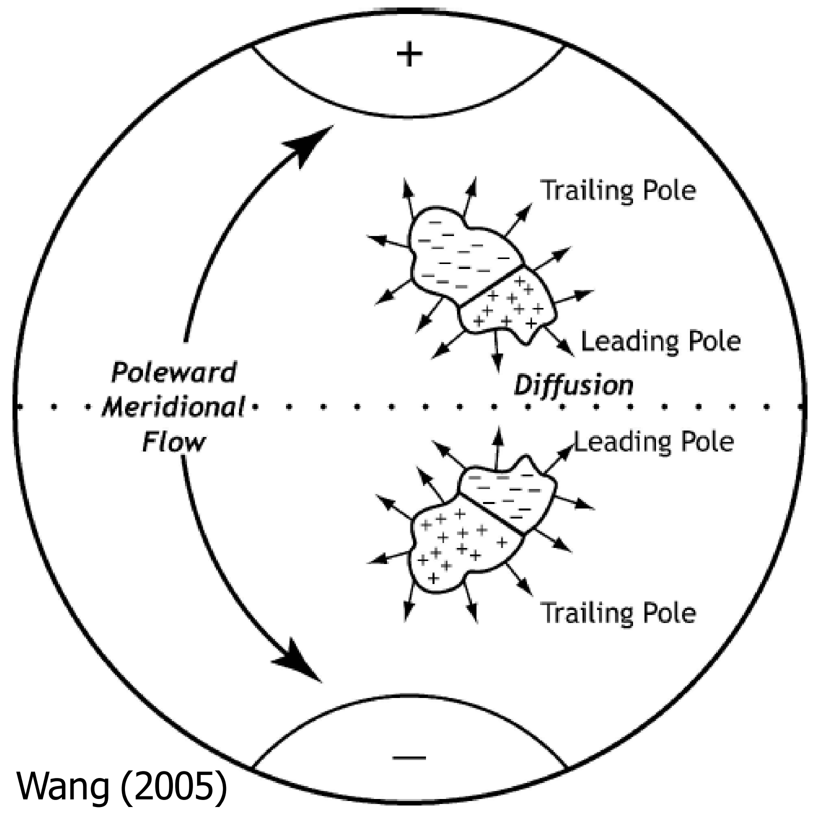
- The BL model appears to capture essential physical processes and can be based to a large degree on observations. Unknown conditions are condensed in a few (3) parameters.

- Long time series (thousands of cycles) and comprehensive parameter studies can be carried out easily.

Babcock-Leighton scenario in a nutshell

Step 1: Surface transport of the emerged magnetic flux contained in systematically tilted bipolar magnetic regions leads to the reversal and buildup of opposite-polarity polar dipole field

Already proposed in Babcock & Babcock (1955)…



Wang (2005)

Validation of the Babcock-Leighton scenario: step 1

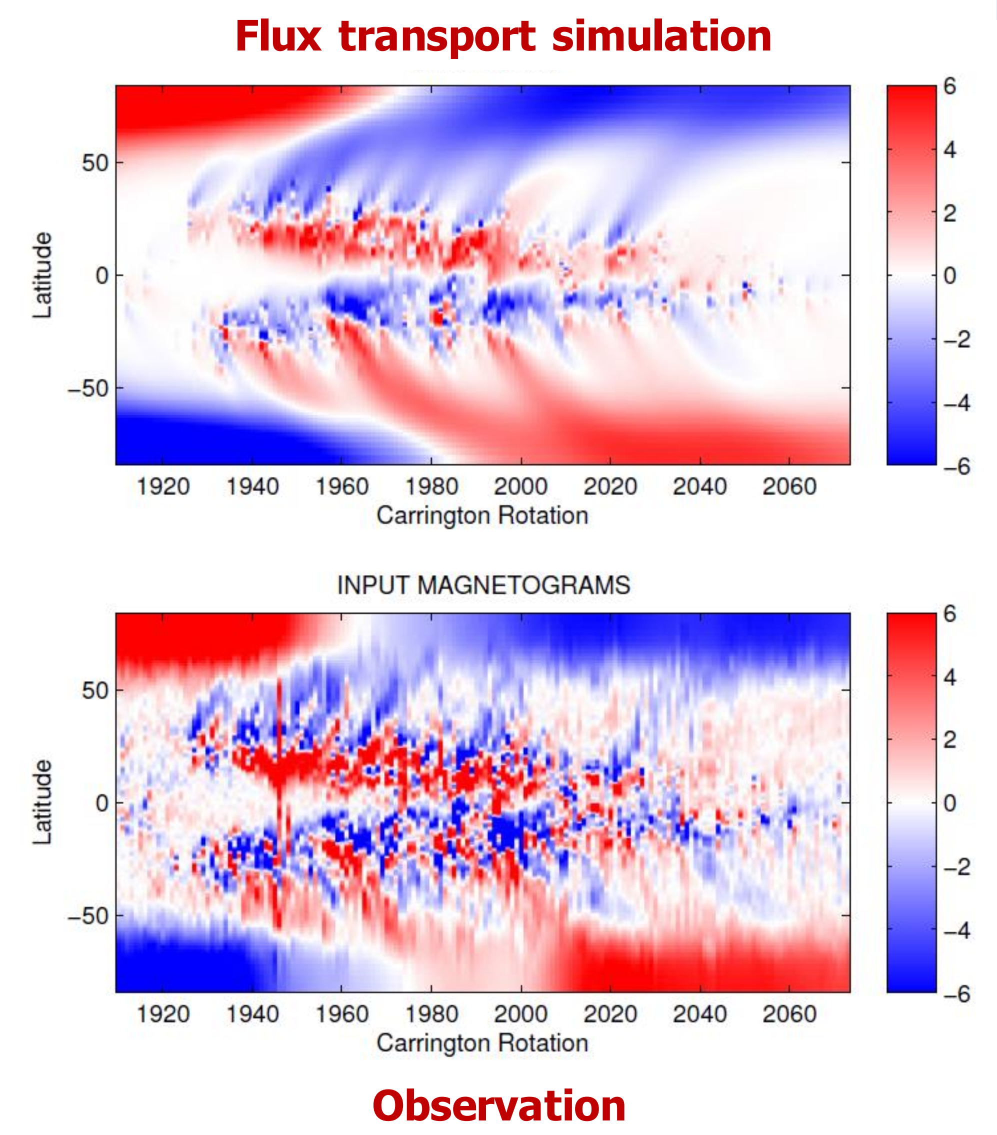
Groundbreaking work
by Y-M. Wang and N.R. Sheeley

Surface flux transport simulations:

- observed flux emergence in **tilted** bipolar magnetic regions
- cancellation & flux advection by diff. rotation, convection, and meridional flow
- polar fields eventually determined by the amount of magnetic flux transported over the equator

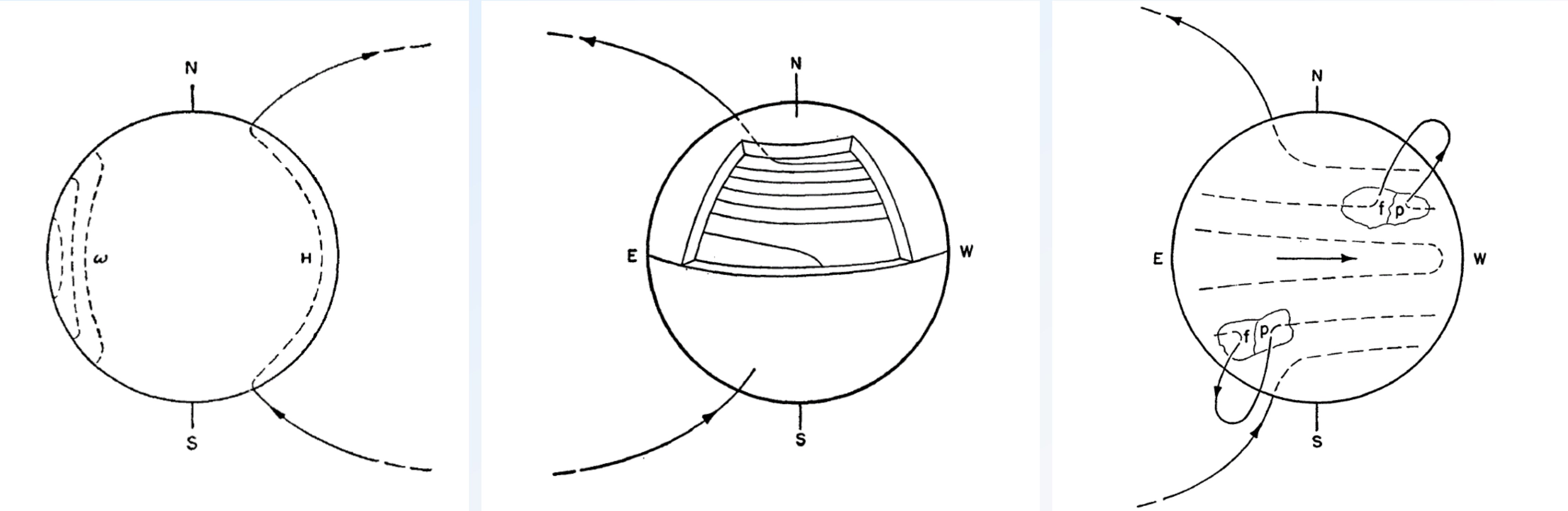
Whitbread et al. (2017)

Time-latitude diagrams of B_r @ surface



Babcock-Leighton scenario in a nutshell

Step 2: The poloidal magnetic flux connected to the polar field is wound up by latitudinal differential rotation, generating the toroidal field whose subsequent emergence produces tilted bipolar magnetic regions (sunspot groups).



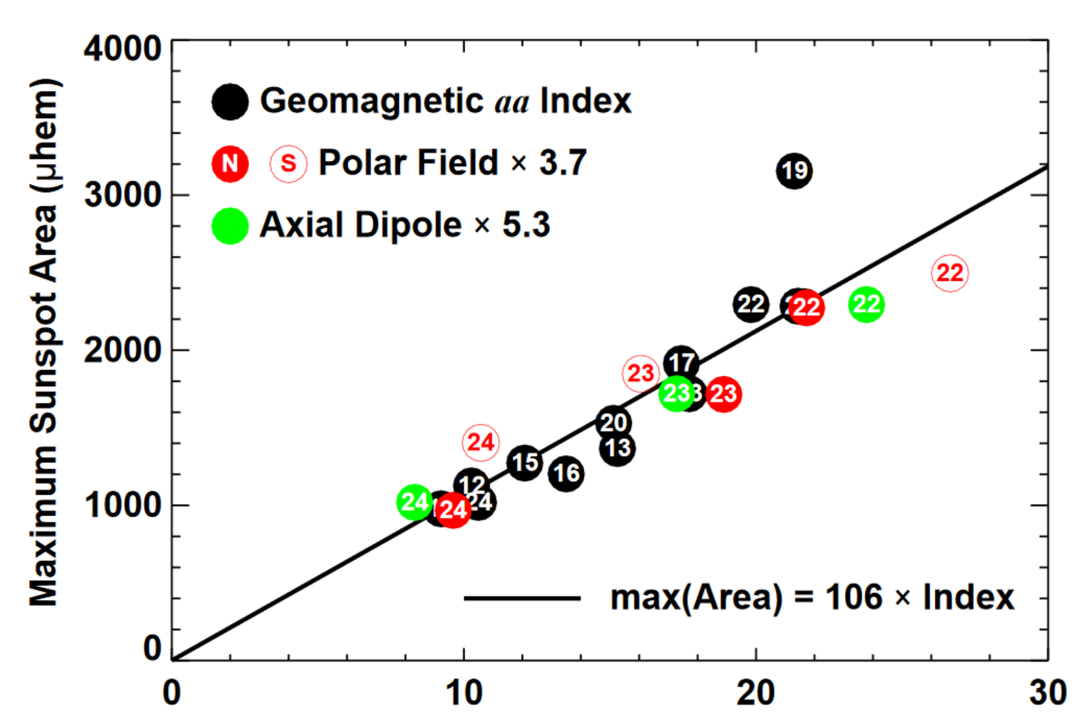
Babcock (1961)

Validation of the Babcock-Leighton scenario: step 2

The strength of a cycle is correlated with the amplitude of the polar fields at the end of the previous cycle.

But: Correlation does not imply causation…
Polar field and „poloidal field of the dynamo"
in principle could be different, but produced
by the same (hidden) process.

Strength of next cycle



Hathaway & Upton (2016)

Solar polar field during activity minimum (proxy)

The crucial question

Q: What is the relevant poloidal flux for the solar dynamo?

Hale's polarity laws → **large-scale toroidal field of fixed orientation** in each hemisphere during a cycle.

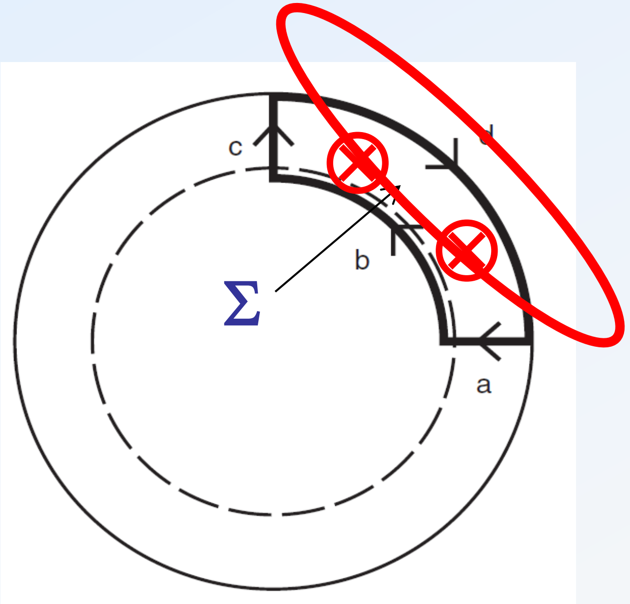
⇨ Need to consider the **net toroidal flux** in a hemisphere, determined from the azimuthally averaged induction equation

Determine toroidal flux in a hemisphere: integrate induction equation over a meridional surface Σ and apply **Stokes' theorem**

Consider $\int_{\delta\Sigma}(\mathbf{U}\times\mathbf{B})\cdot\mathbf{dl}$

Only significant contribution: surface part

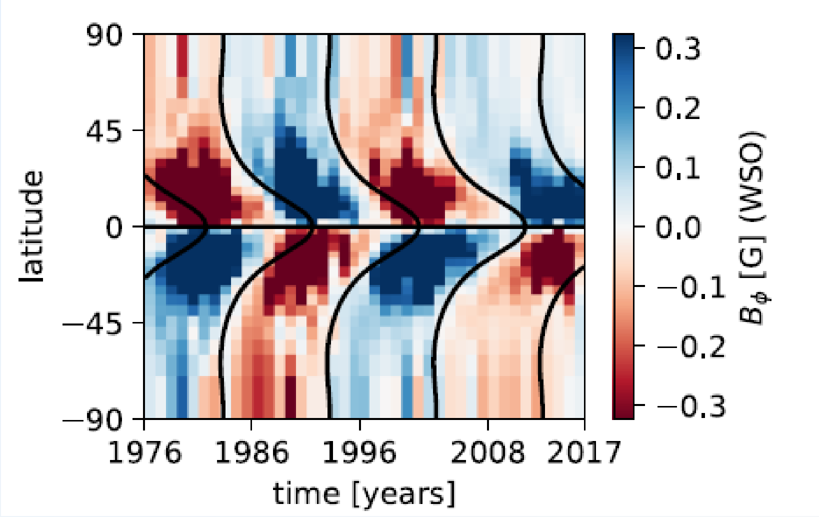
$$\frac{d\Phi^{N}_{tor}}{dt} = \int_0^1 (\Omega - \Omega_{eq}) B_r R_\odot^2 \, d(\cos\theta)$$



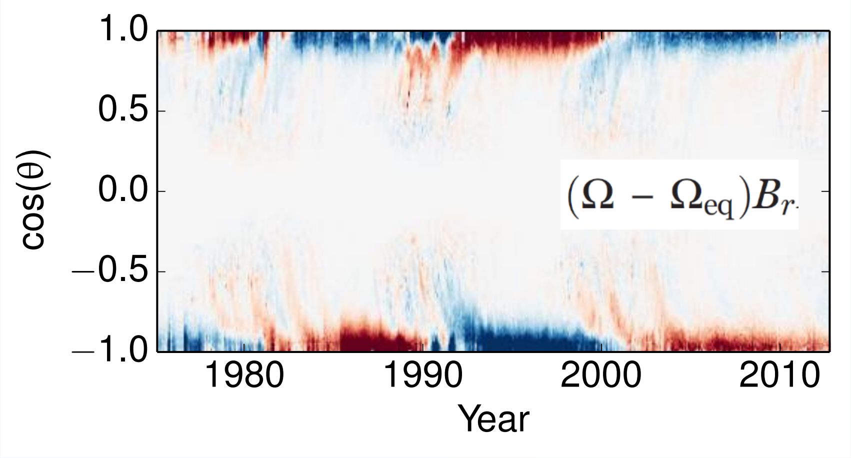
Meridional cut

Cameron & S.
Science **347**, 1333 (2015)

Bϕ at solar surface



Cameron et al. (2018)



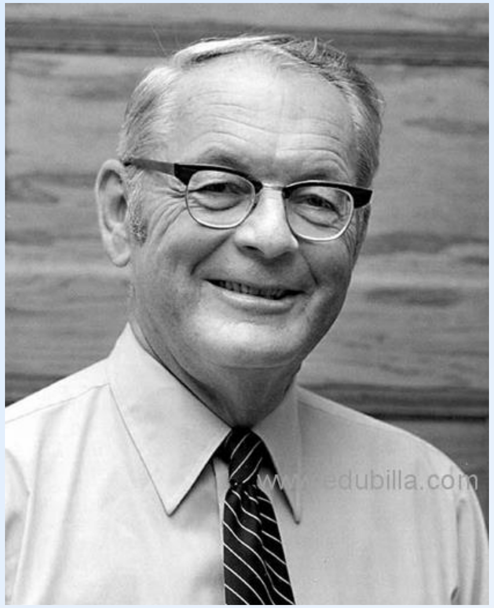
Strongly dominated by polar fields...

Validation of the Babcock-Leighton scenario: step 2

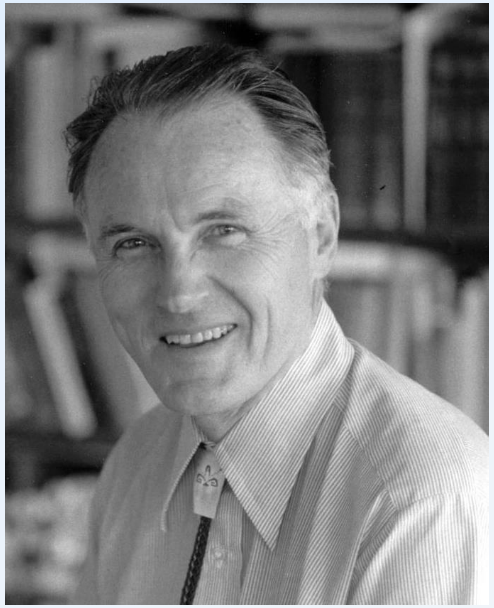
Q: What is the relevant poloidal flux for the solar dynamo?

A: The magnetic flux connected to the polar field represents the dominating poloidal source of the net toroidal flux which emerges in the subsequent cycle.

Any other poloidal field (hidden in the convection zone) leads to equal amounts of positive and negative toroidal flux and thus does not contribute to the net toroidal flux required by Hale's polarity laws.



Horace W. Babcock
(1912-2003)



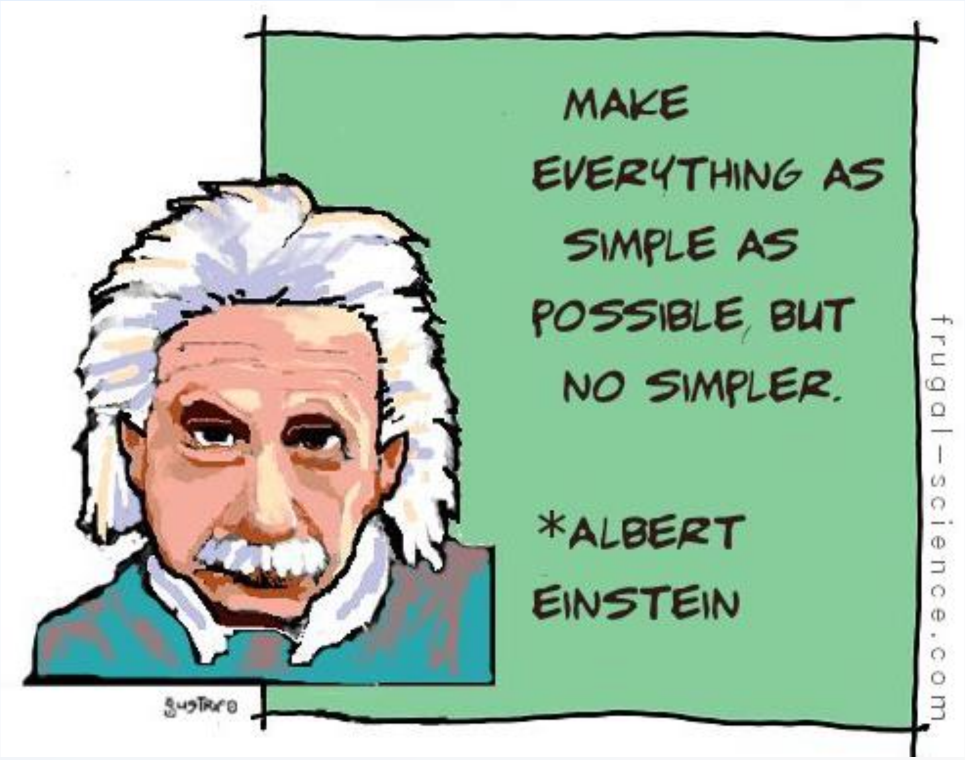
Robert B. Leighton
(1919-1997)

The Babcock-Leighton model seems to capture essential features of the large-scale solar dynamo.

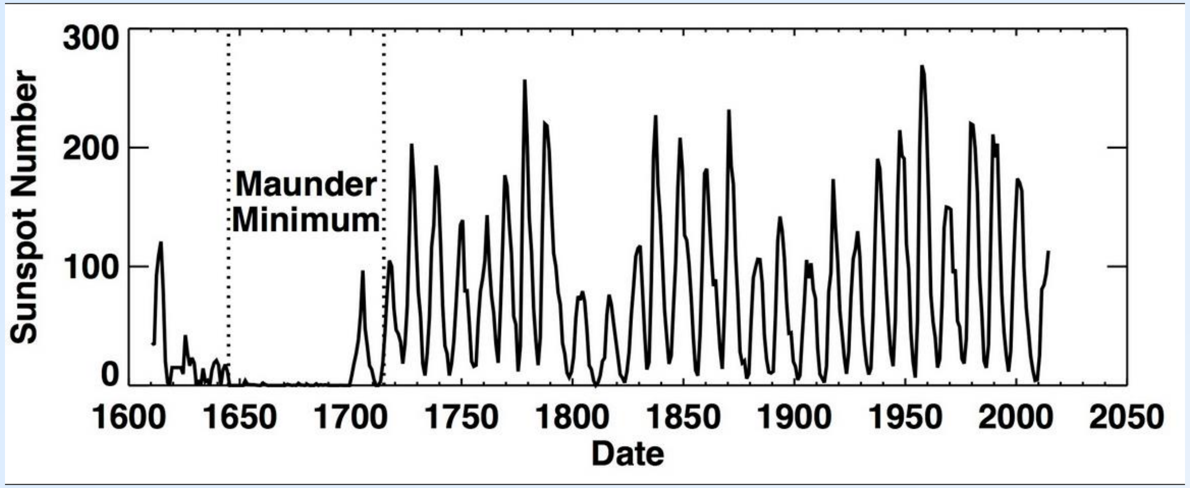
Update of the BL model taken account of the observational results obtained since the 1960s: Cameron & S. (2017, A&A)

Outline

- A brief history
- Challenges to the „current paradigm"
- Babcock-Leighton redux
- **Cycle variability**



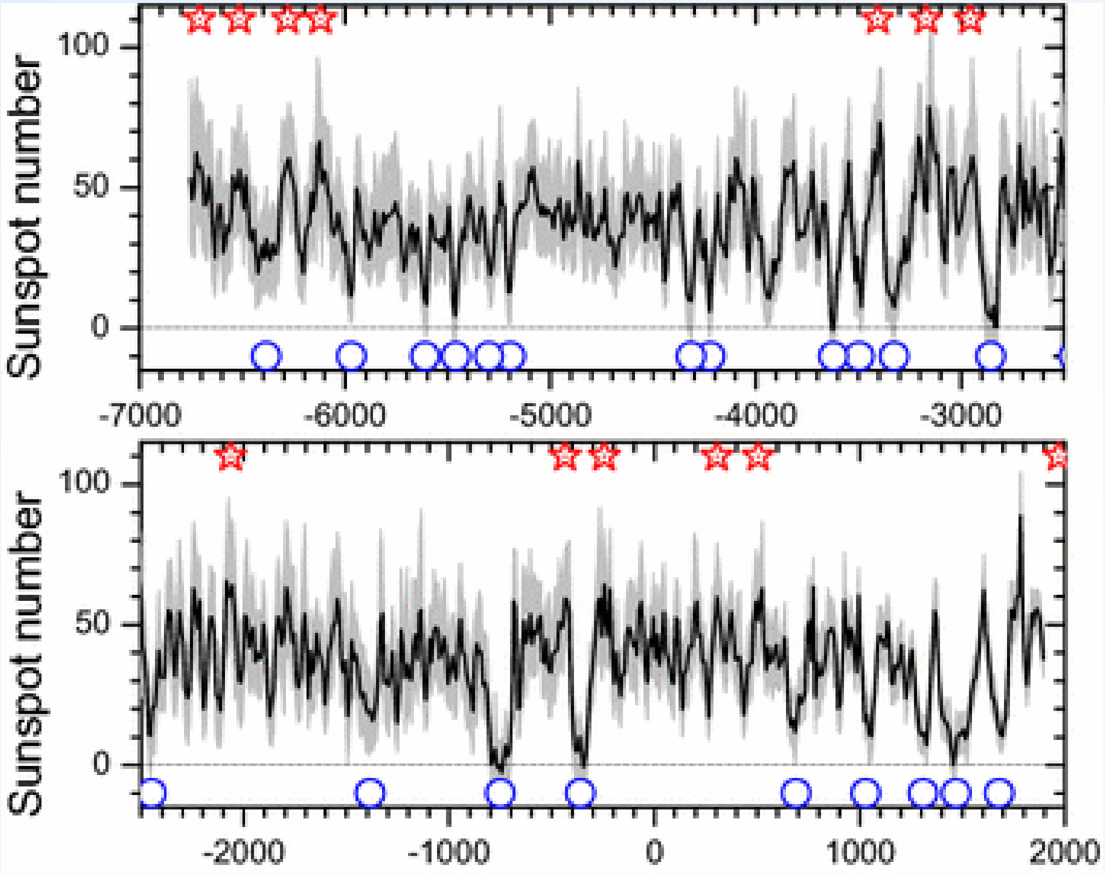
Cycle variability



Considerable cycle-to-cycle variability with occasional „grand" minima and maxima

- nonlinear effects?
- intermittency?
- **stochastic fluctuations?**

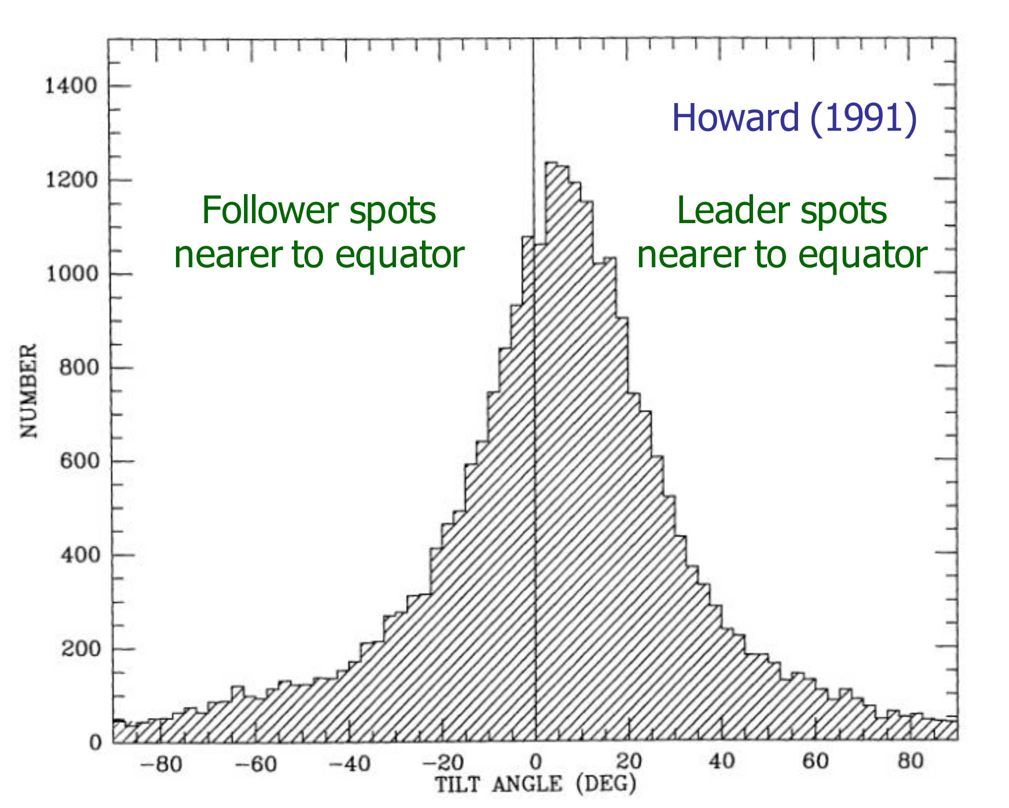
Sunspot numbers during the holocene as inferred from cosmogenic isotopes (^{10}Be, ^{14}C)



Years BP

(Usoskin et al., 2016)

Randomness matters



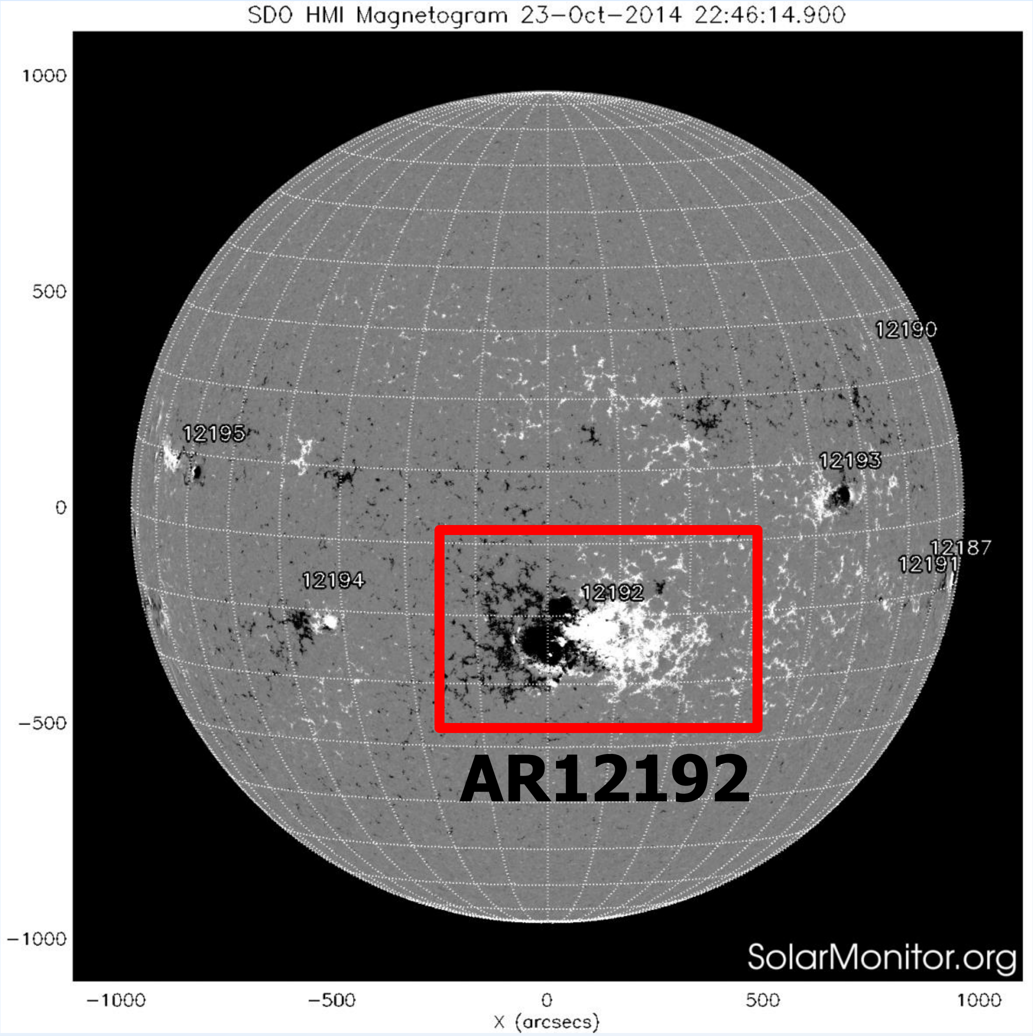
Histogram of sunspot group tilt angles
(Mt. Wilson, 1917 – 1985)

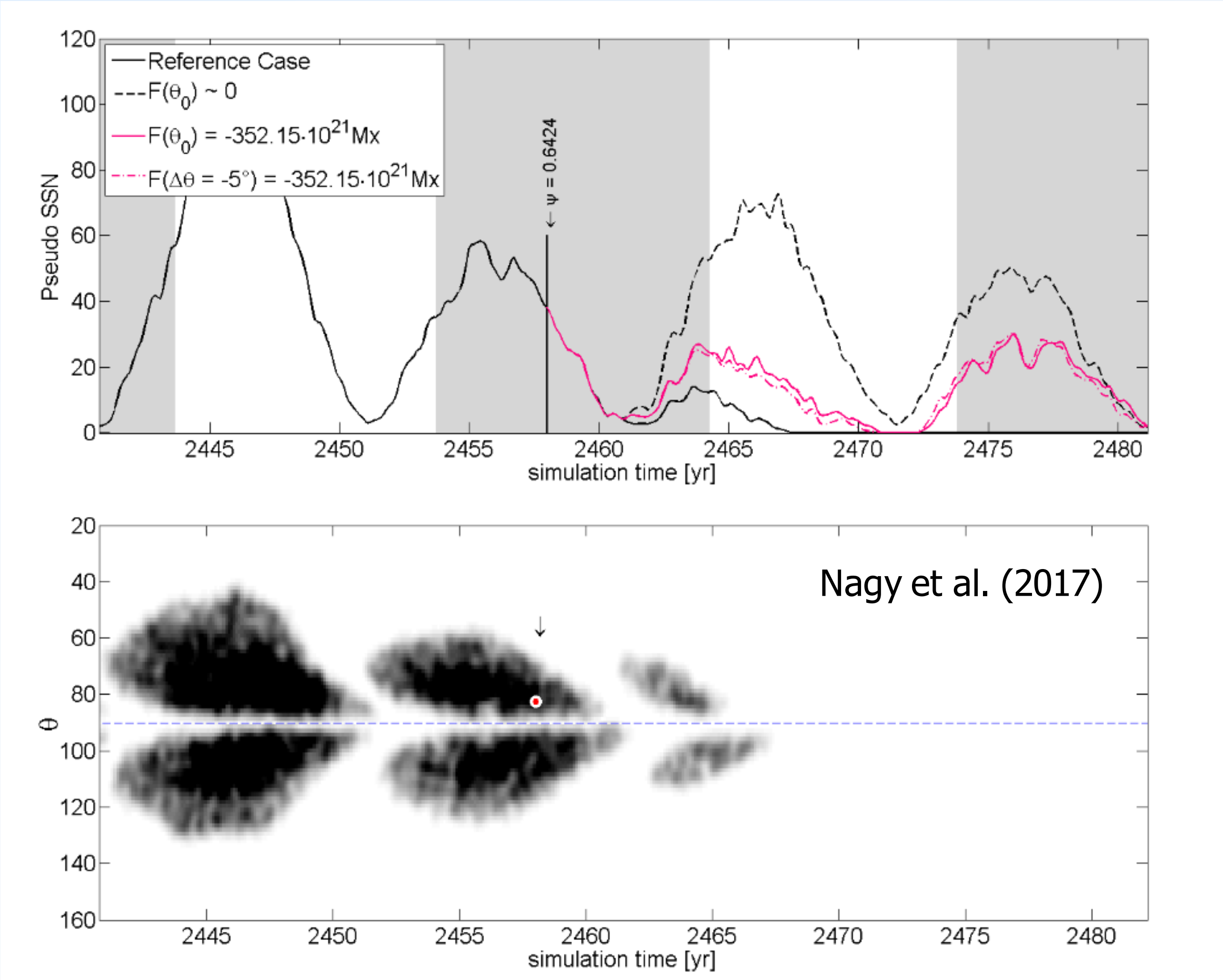
Substantial **scatter** of sunspot group tilt angles

A single large bipolar region carries an amount of magnetic flux comparable to that contained in the polar field.

The weakness of cycle 24 can be understood as the effect of a few active regions with „wrong" tilt (Jiang et al., 2015)

Big bipolar regions contain a lot of magnetic flux…





October 2014

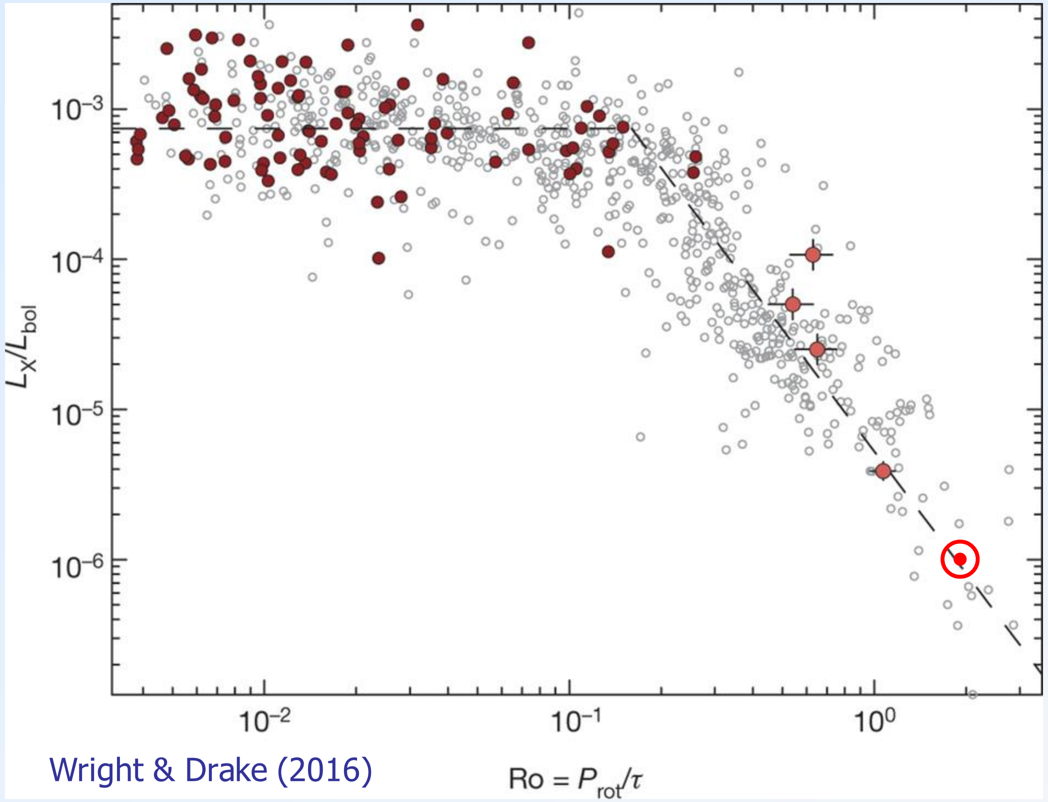
The spot that killed the dynamo….

Simplicity: one step further

The Sun is not a particularly active star…

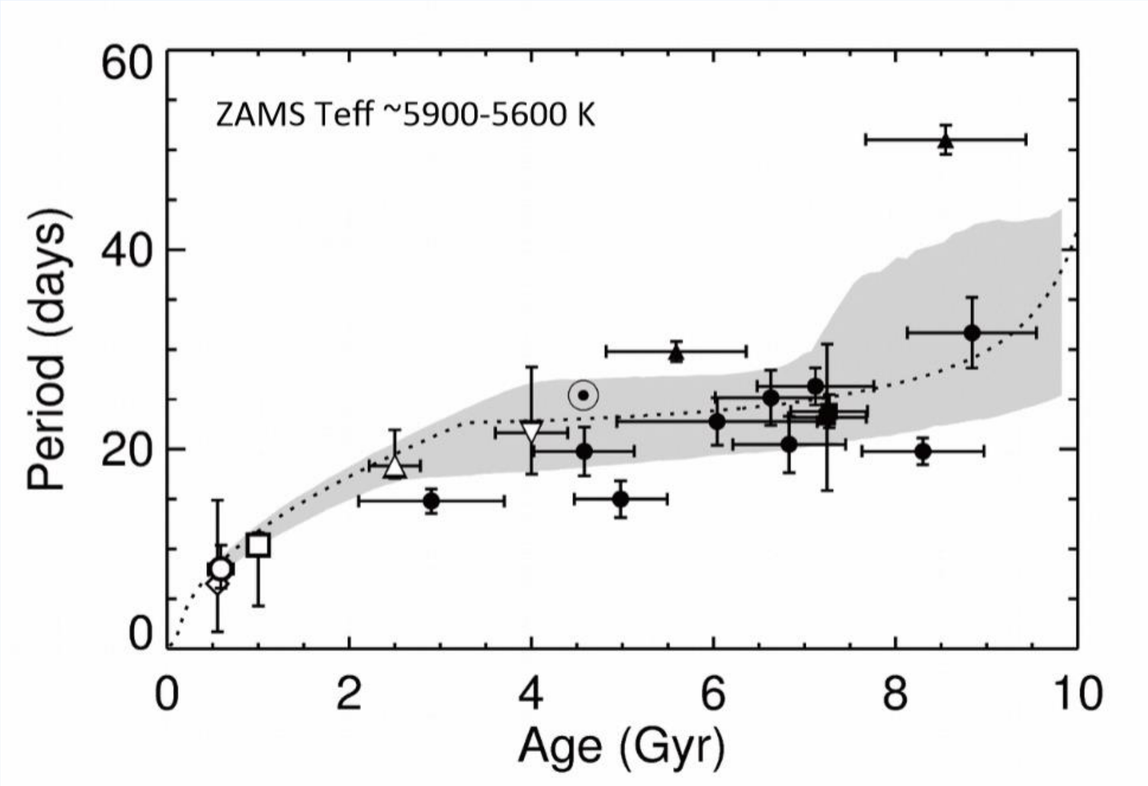
Actually, the slowly rotating Sun appears to be near marginal cyclic dynamo excitation
(van Saders et al., 2016; Metcalfe et al., 2016
Olspert et al., 2018)

↑
Activity



Wright & Drake (2016)

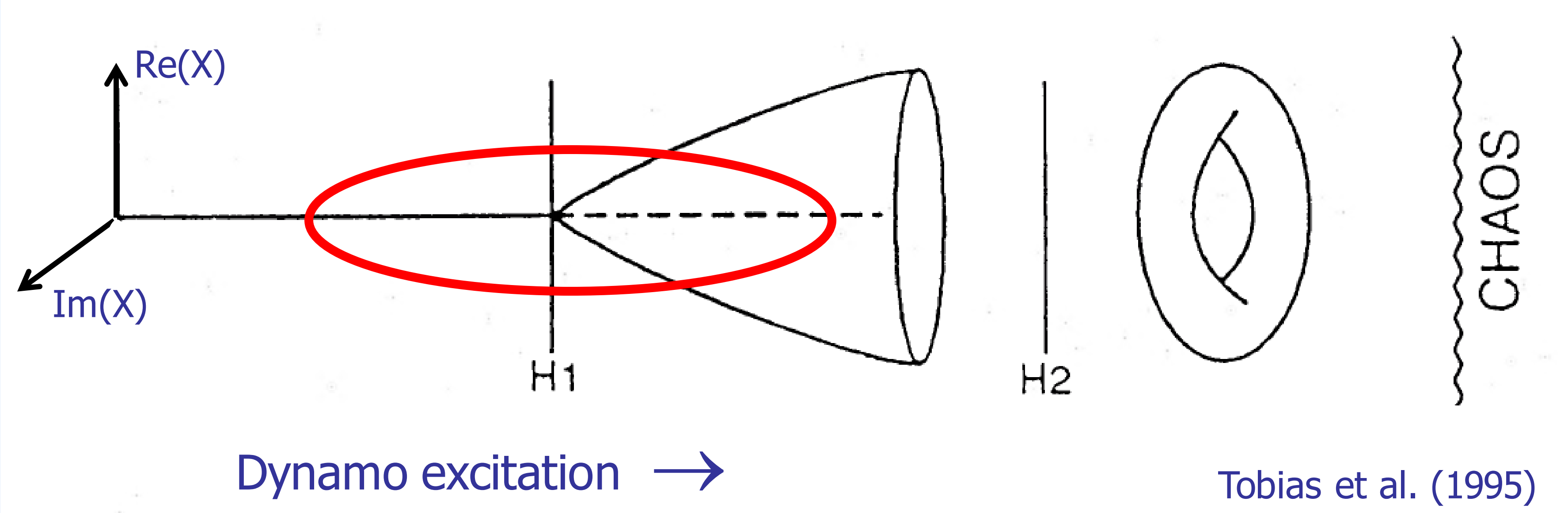
← Rotation rate



J. van Saders

Simplicity: one step further

Models for oscillatory dynamos typically exhibit a Hopf bifurcation at critical dynamo excitation: a fixed point becomes unstable and spawns a limit cycle (periodic solution)



Generic normal form near a Hopf bifurcation

Normal form
$$\frac{\mathrm{d}X}{\mathrm{d}t} - (\beta + i\omega_0)X + (\gamma_r + i\gamma_i)|X|^2 X = 0$$

Linear growth rate β

Amplitude $|X| = \sqrt{\beta/\gamma_r},$

Linear frequency ω_0

Frequency $\omega = \omega_0 - \gamma_i \beta/\gamma_r$

All four parameters are **constrained by observation**:

Mean sunspot number since 1700: $|X| = 64$ for sinusoidal cycles

Recovery from Maunder minimum: $\beta = 1/50 \text{ year}^{-1}$

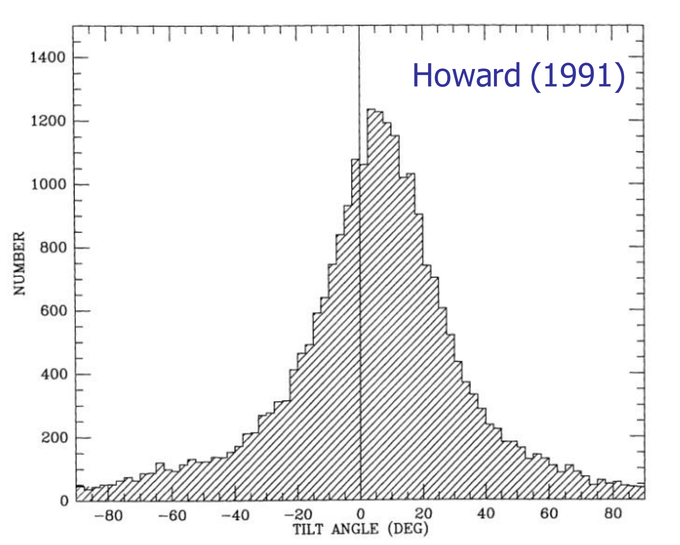
~11-year cycles during Maunder minimum: $\omega = \omega_0 = 2\pi/(22 \text{ yrs})$

Normal form with multiplicative noise

Random forcing of the dynamo owing to **scatter of tilt angles**:
stochastic differential equation

$$\mathrm{d}X = \left(\beta + i\omega_0 - (\gamma_r + i\gamma_i)|X|^2\right) X\mathrm{d}t + \sigma X \mathrm{d}W_c = 0\,,$$

W_c complex Wiener process with variance = 1 after 11 years
(random walk with uncorrelated Gaussian increments)



Histogram of sunspot group tilt angles
(Mt. Wilson, 1917 – 1985)

Noise amplitude: $\sigma = 0.40$

from polar field variability due to observed tilt angle scatter
($\rightarrow$ consistent with variability of cycle maxima since 1700)

Performed Monte-Carlo simulations with Euler-Maruyama method

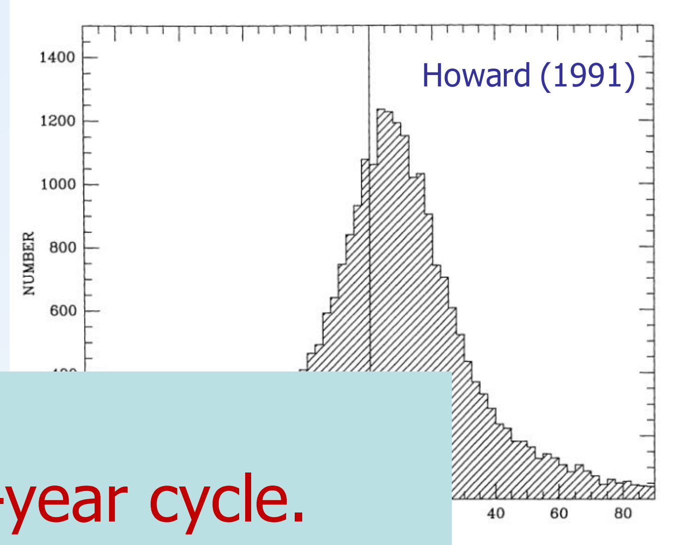
Take Re(X) as a proxy for sunspot number (toroidal flux): $\mathrm{SSN} = |\Re(X)|$

Normal form with multiplicative noise

Random forcing of the dynamo owing to **scatter of tilt angles**:

stochastic differential equation

$$\mathrm{d}X = \left(\beta + i\omega_0 - (\gamma_r + i\gamma_i)|X|^2\right) X\mathrm{d}t + \sigma X \mathrm{d}W_c = 0$$



oup tilt angles

1985)

W_c

Noise amp

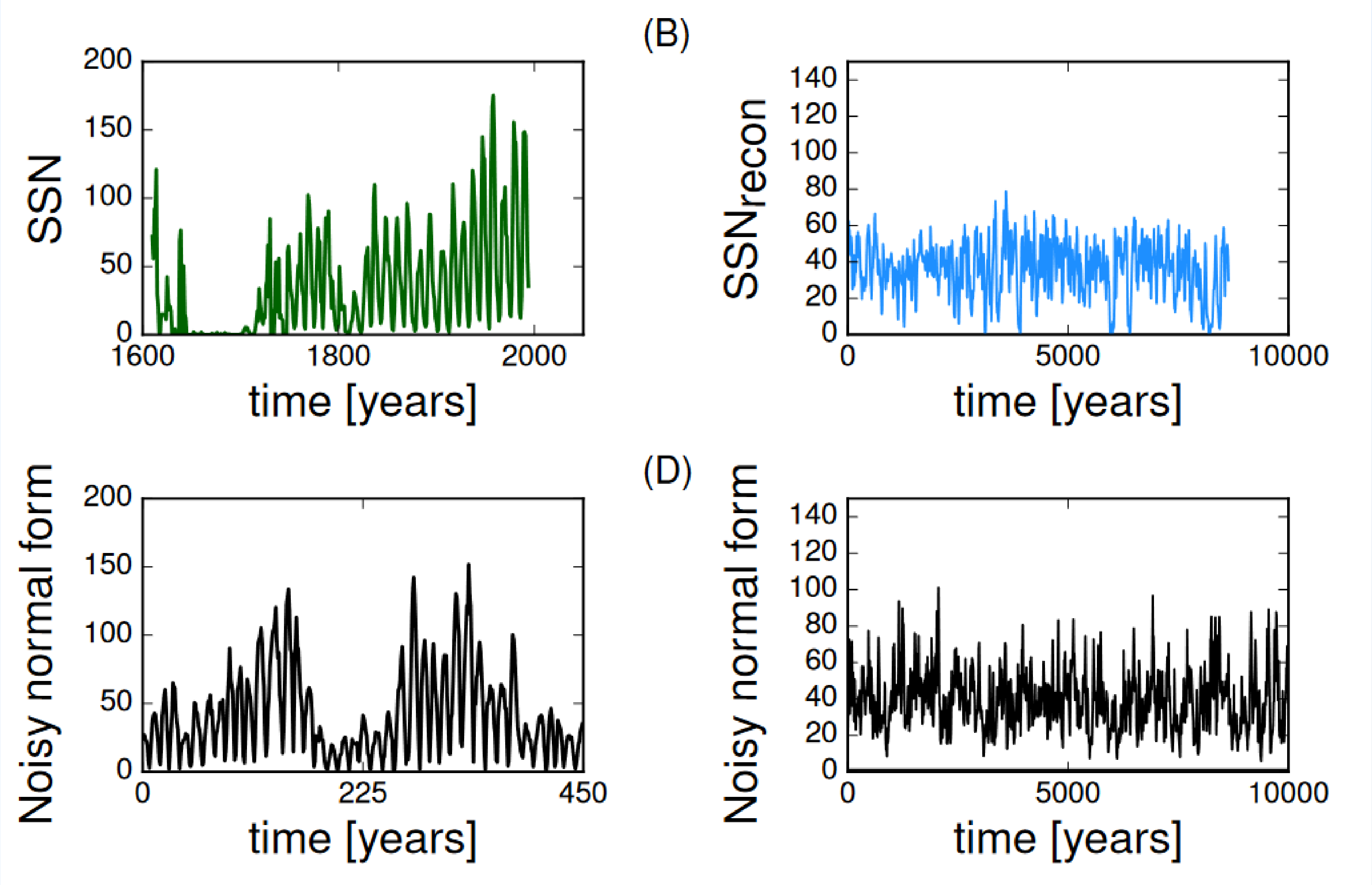
Performed

Take Re(X) as a proxy for sunspot number (toroidal flux): $\mathrm{SSN} = |\Re(X)|$

No intrinsic periodicities apart from the basic 11-year cycle.

May thus serve as a proper null case for evaluating the significance of periodicities found in the empirical record.

Sunspot number vs. normal form



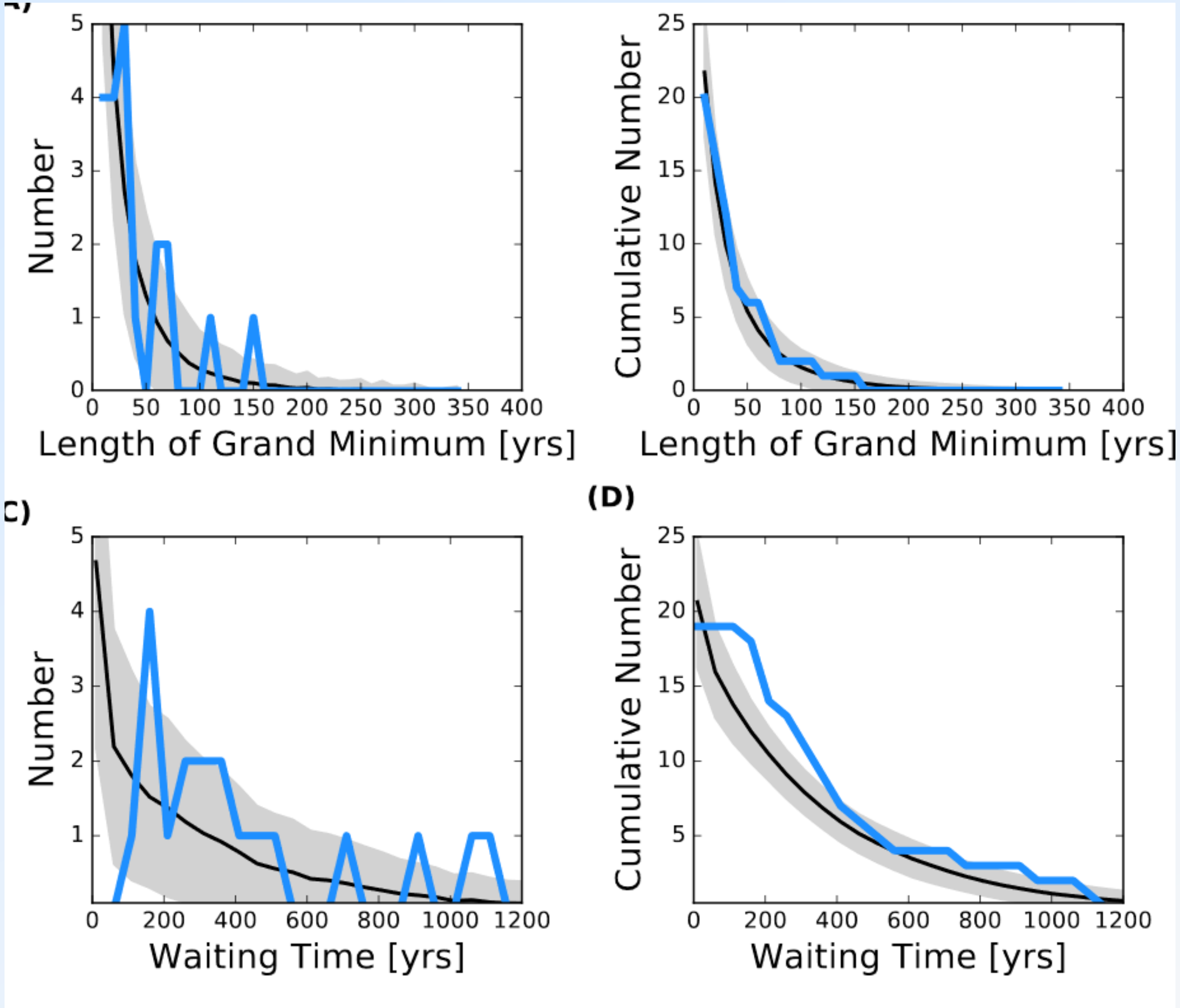
Empirical sunspot numbers

left: direct observations

right: inferred from cosmogenic isotopes (^{10}Be, ^{14}C)

Noisy normal-form model
(one realization)

Statistics of grand minima: data vs. model



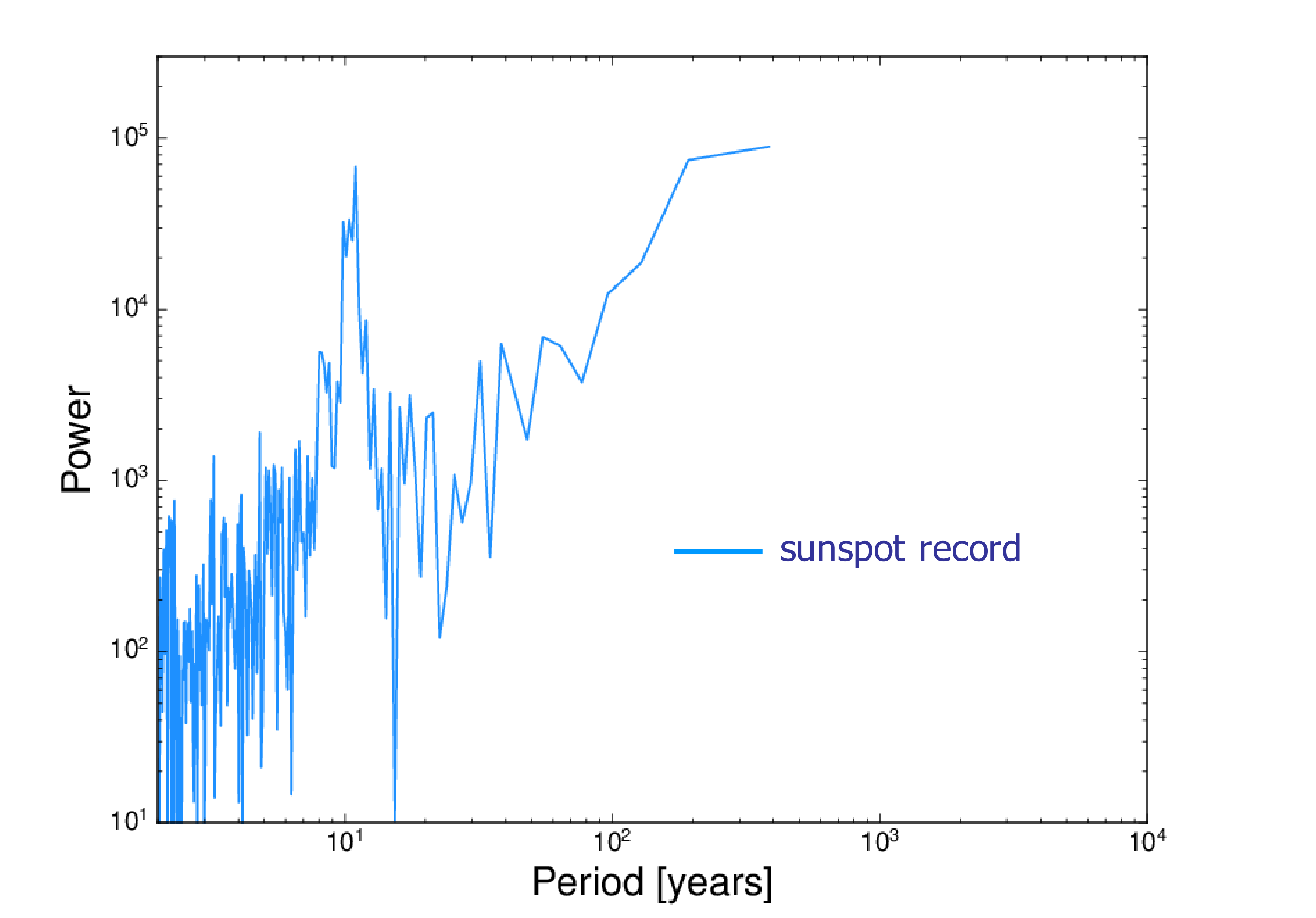
Usoskin et al. (2016)
(cosmogenic isotopes)

Normal-form model
(1000 realizations of
10,000 years each)

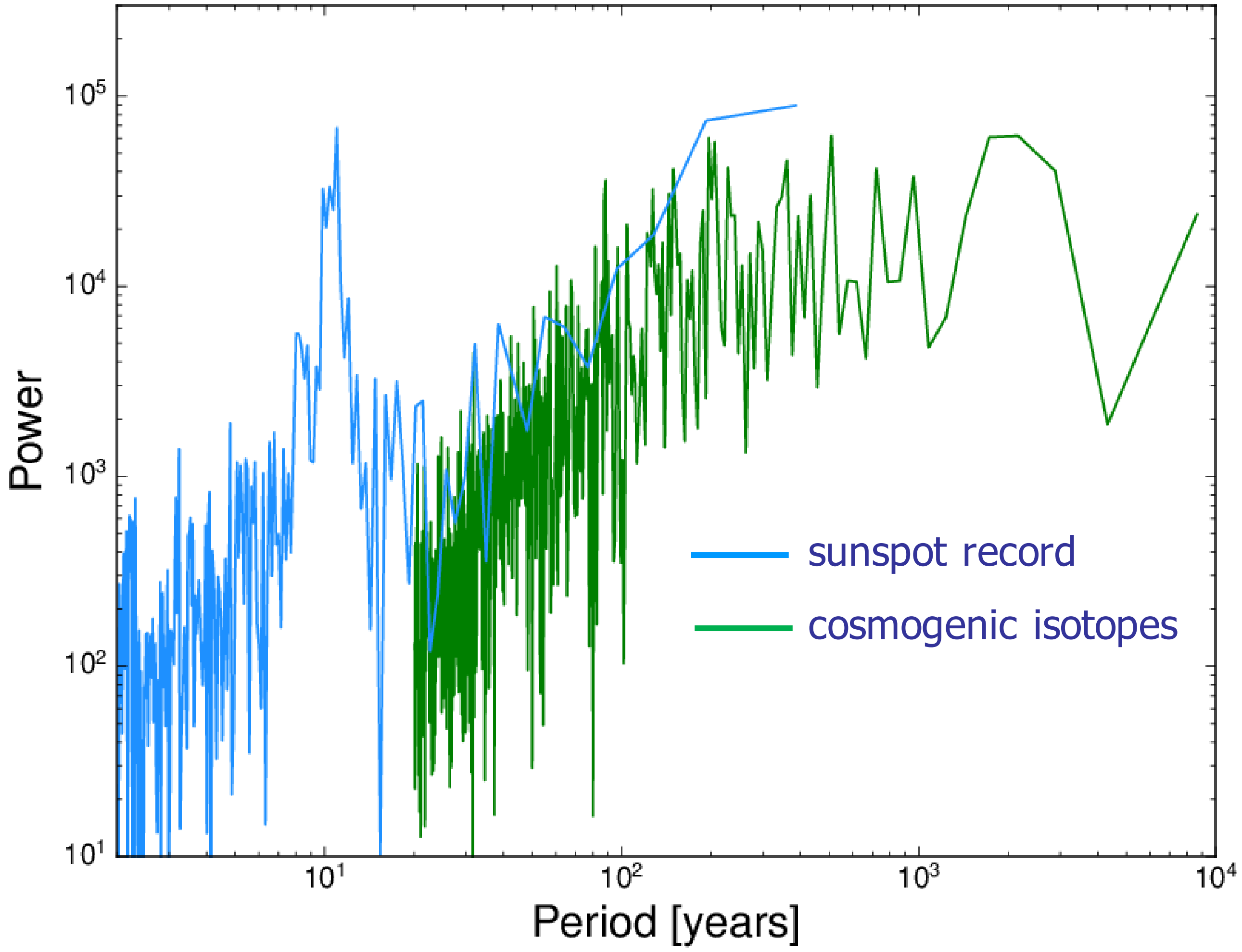
standard deviation

Exponential distributions are consistent with a Poisson process.

Power spectra: sunspot numbers



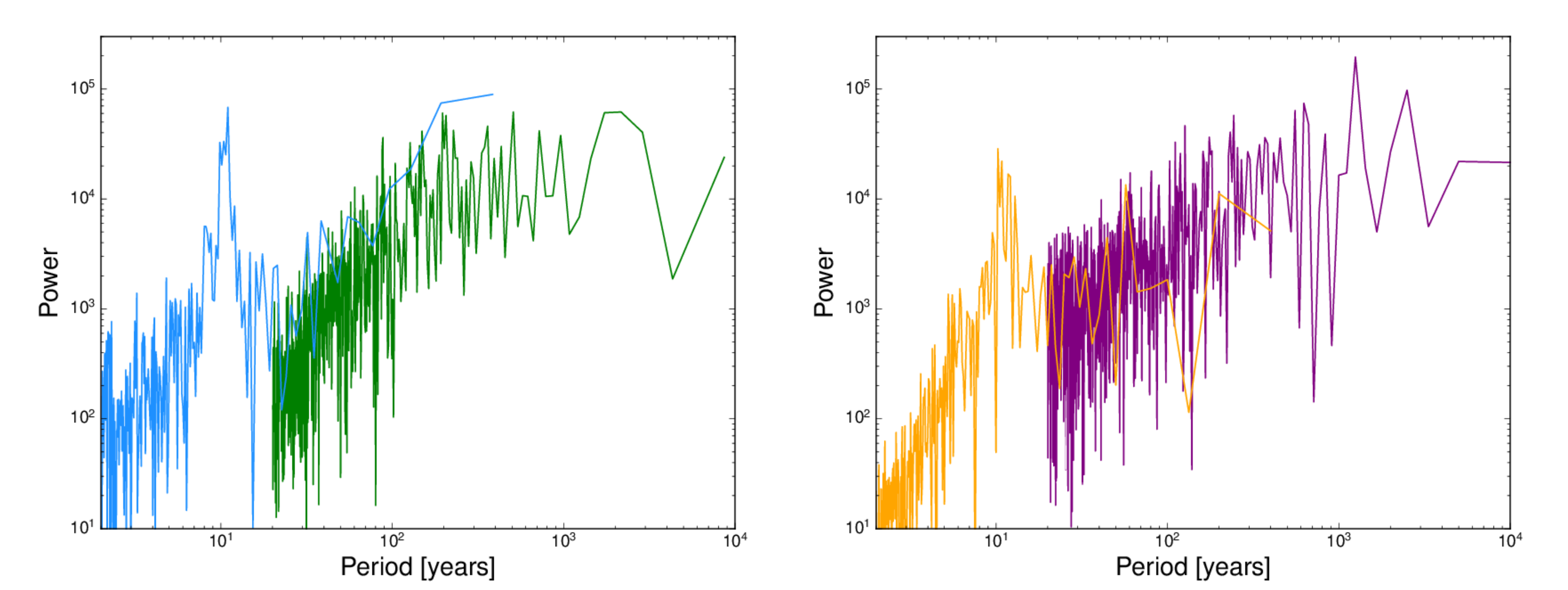
Power spectra: sunspot numbers



Power spectra: SSN vs. normal form

Observation

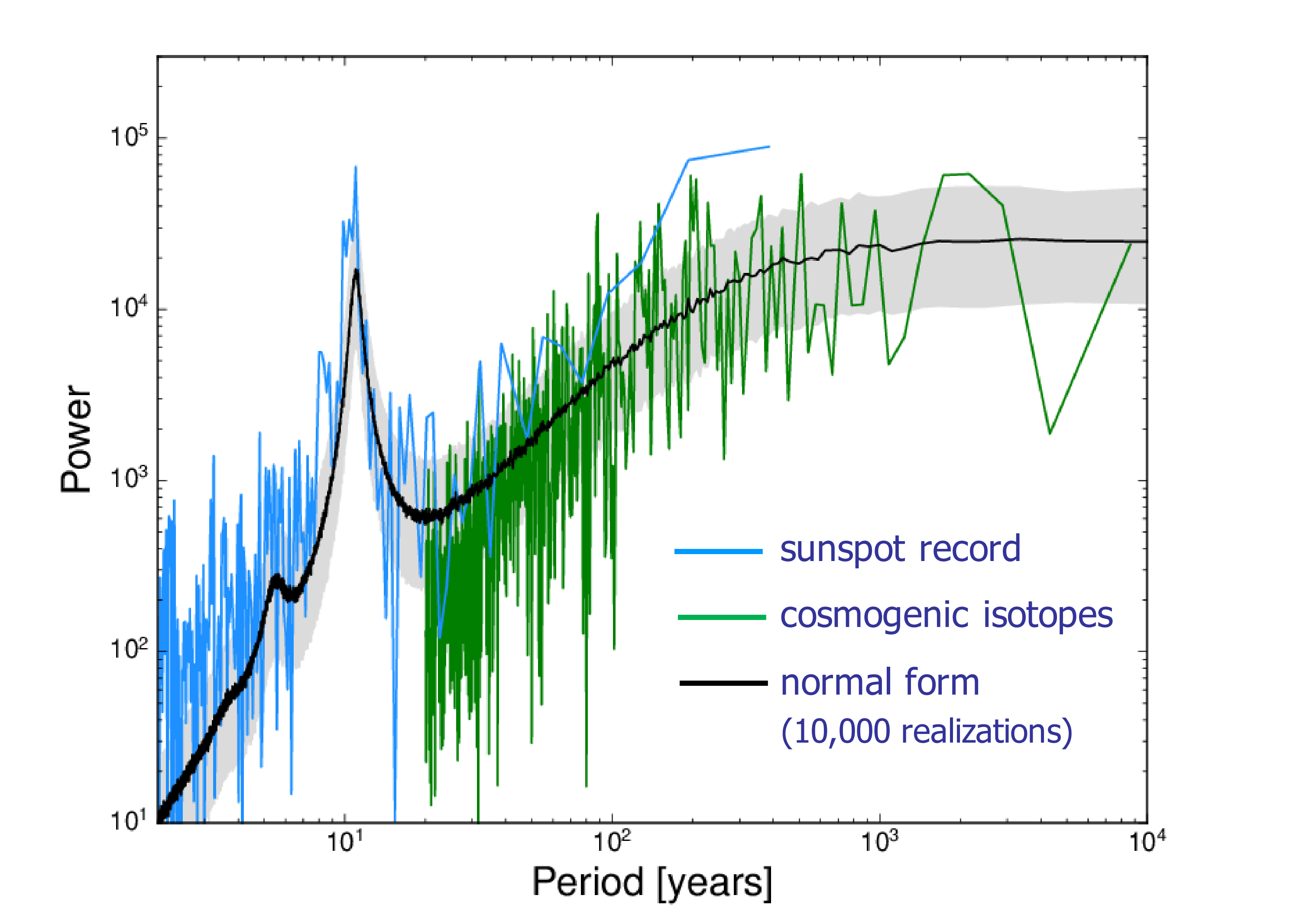
NF: one realisation



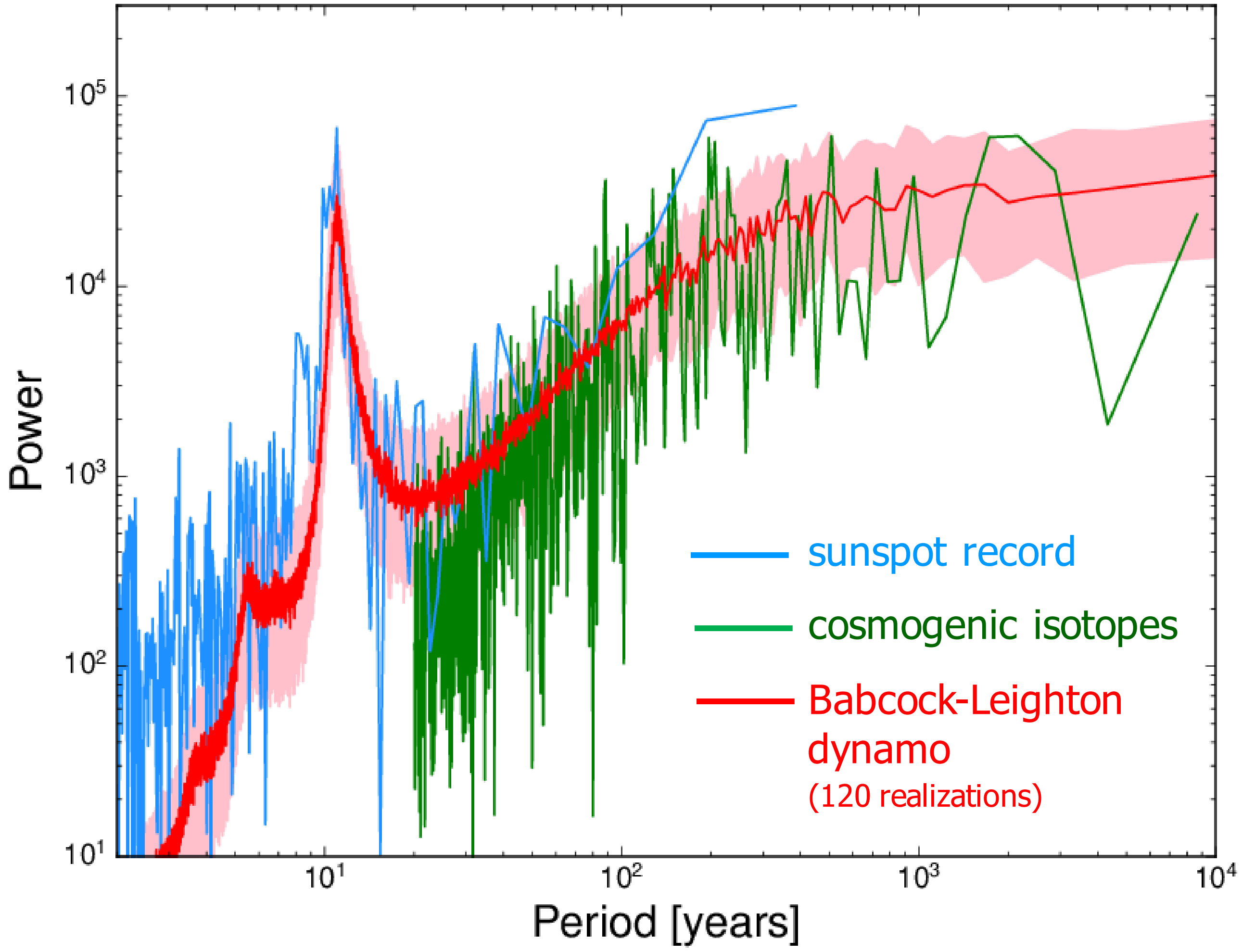
sunspot record
cosmogenic isotopes

normal form (350 yrs)
normal form (10,000 years)

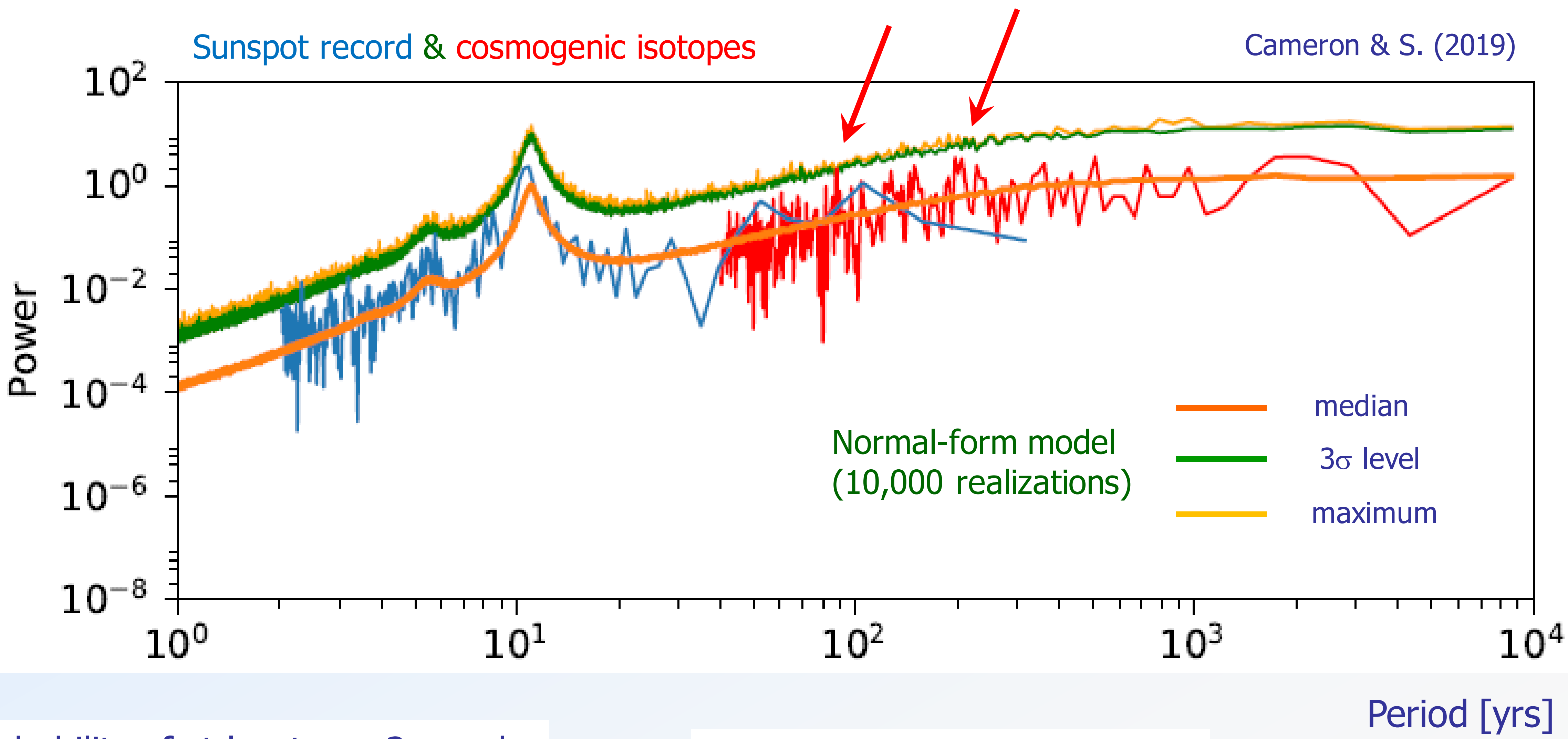
Power spectra: SSN vs. normal form



Power spectra: SSN vs. BL dynamo



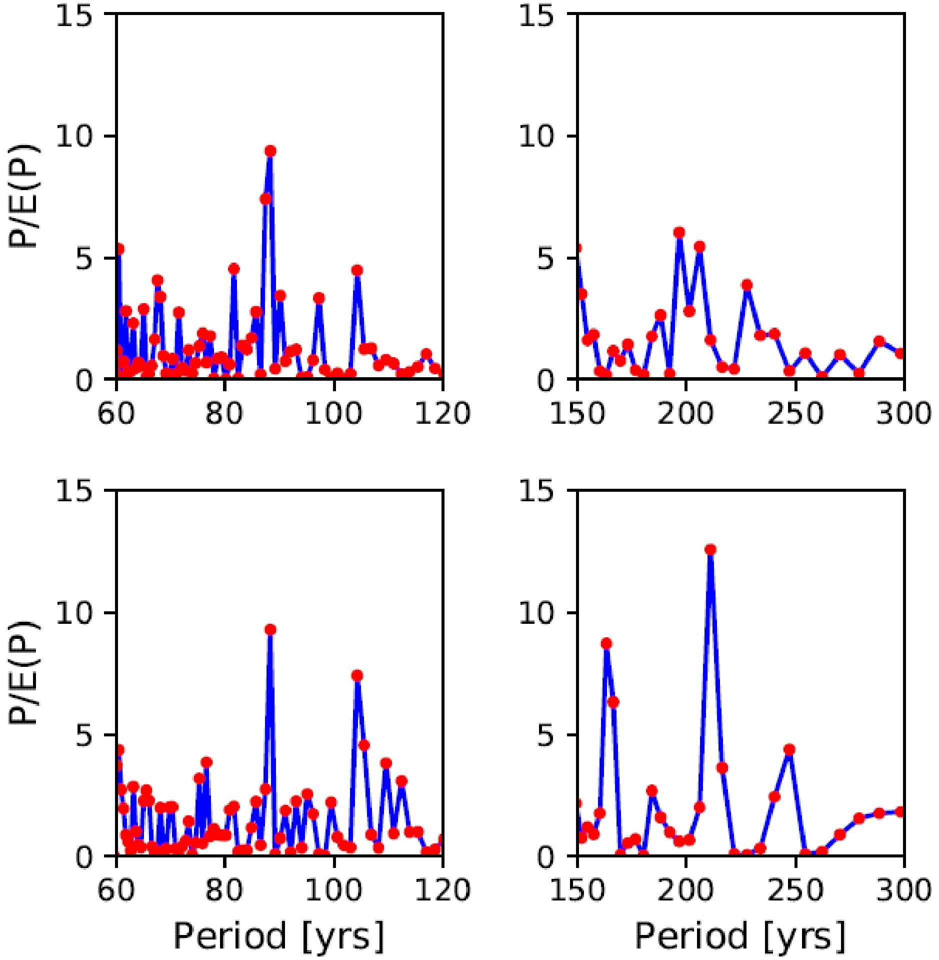
The significance of single peaks



Probability of at least one 3σ peak in 216 resolved frequency bins → $p = 1 - 0.99865^{216} = 0.253,$

The significance of single peaks

Gleissberg & de Vries peaks from cosmogenic isotopes



3σ peaks from realizations
of the noisy normal-form model

My summary message…

- Scarce observational information about convection zone and on the conditions in other stars…

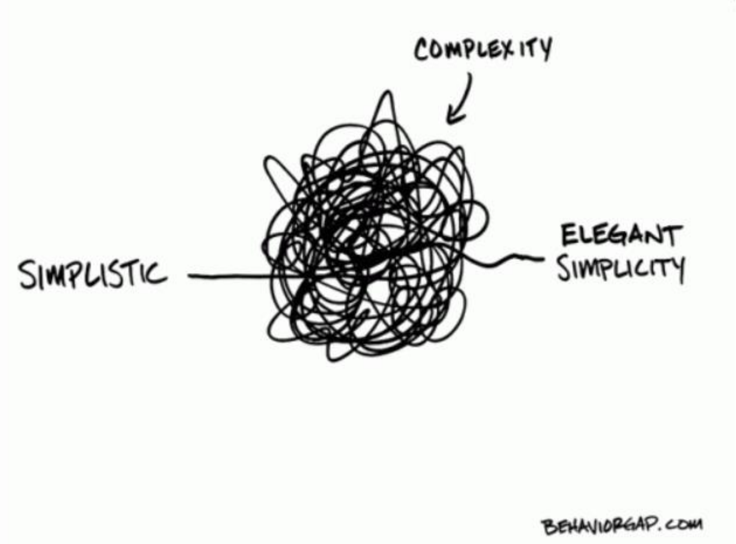
- … suggests wide range of approaches between

  $\rightarrow$ **back to the roots** (BL or even simpler)
  lumps unknown properties into a few parameters

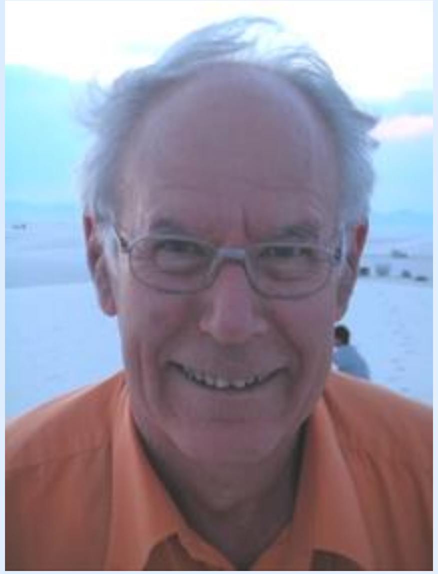
  … as well as …

  $\rightarrow$ **up to the treetop** (3D MHD)
  quantitative understanding of basic processes

- Cycle variability consistent with random fluctuations
  $\rightarrow$ limited scope for predictions

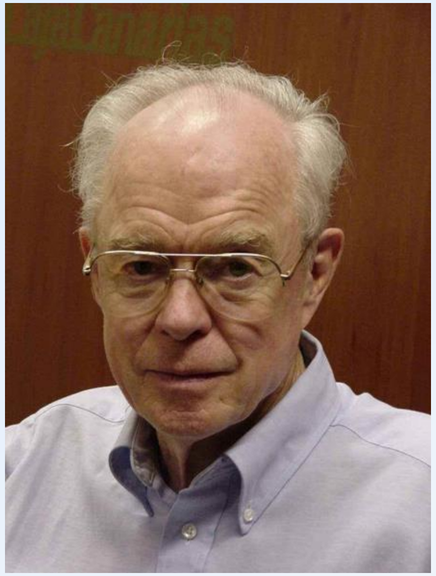


In lieu of a conclusion…



„Many suggestive models illuminate various aspects of the solar cycle; but details are frequently obscure and more comprehensive calculations have still to be completed."

N. O. Weiss (1971)



„The shifting nuances of observation have many times in the past sunk a substantial theoretical ship, and the most likely explanation of today may be found washed up on the beach tomorrow."

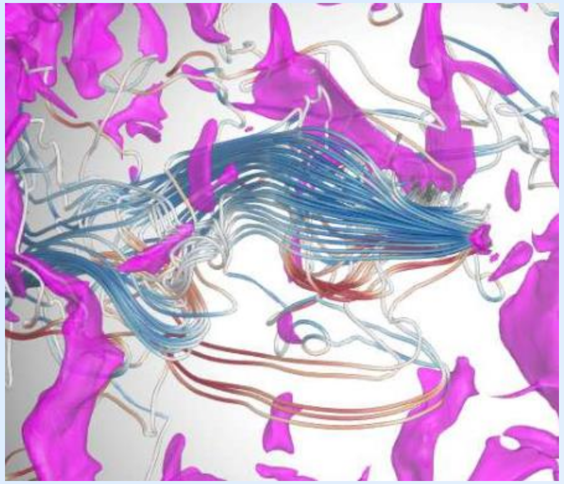
E. N. Parker (1989)

The realm of the unknown

- turbulent magnetic diffusity
- spatial structure and temporal variability of the meridional circulation
- large-scale convective patterns (the „convection conundrum”)
- strength of convective pumping
- maintainance of the tachocline within the convection zone
- why does the sun rotate solar-like
- why does flux emerge in the way it does
- how does small-scale dynamo action affect the large-scale dynamics
- size and properties of the overshoot/subadiabatic layer
- penetration of flows and field into the radiative zone

Back to the future: complexity redux

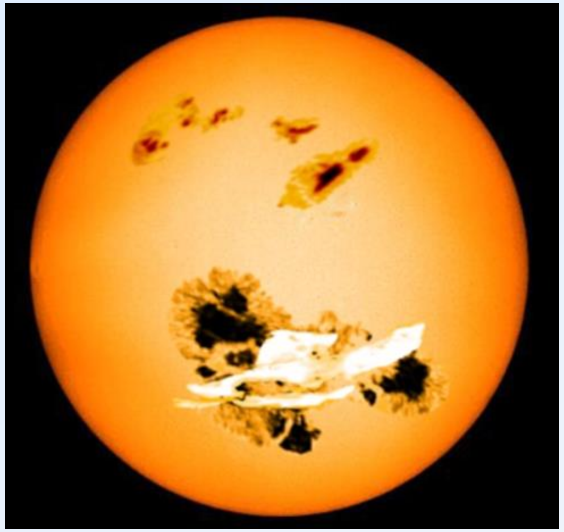
Chen et al. (2017)



Sun:

→ flux emergence in tilted bipolar magnetic regions is crucial, determines of the excitation of the dynamo;

→ the connection between the subsurface toroidal field and flux emergence seems to be highly complex and non-trivial.



Other magnetically active stars:

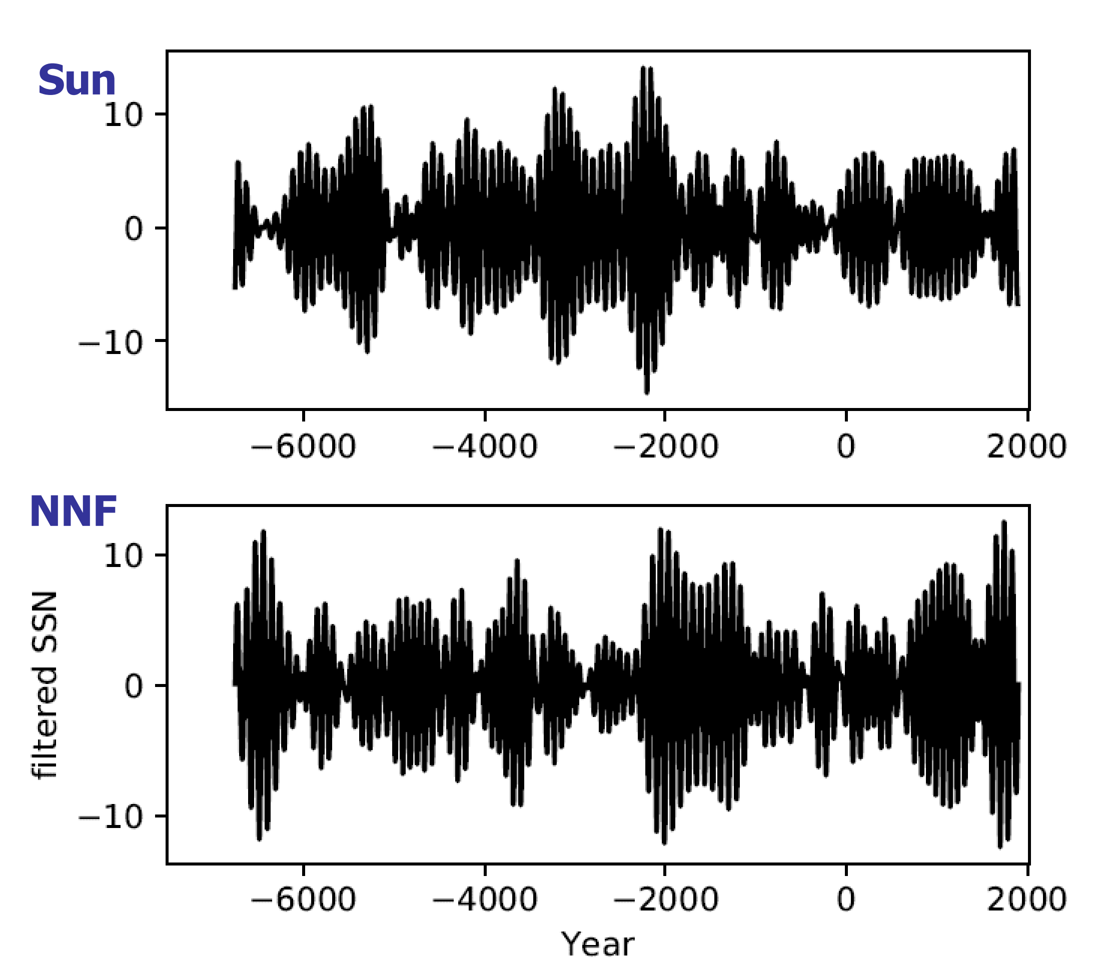
internal differential rotation, convective flows, meridional flows, tilt angles…

→ mostly unknown

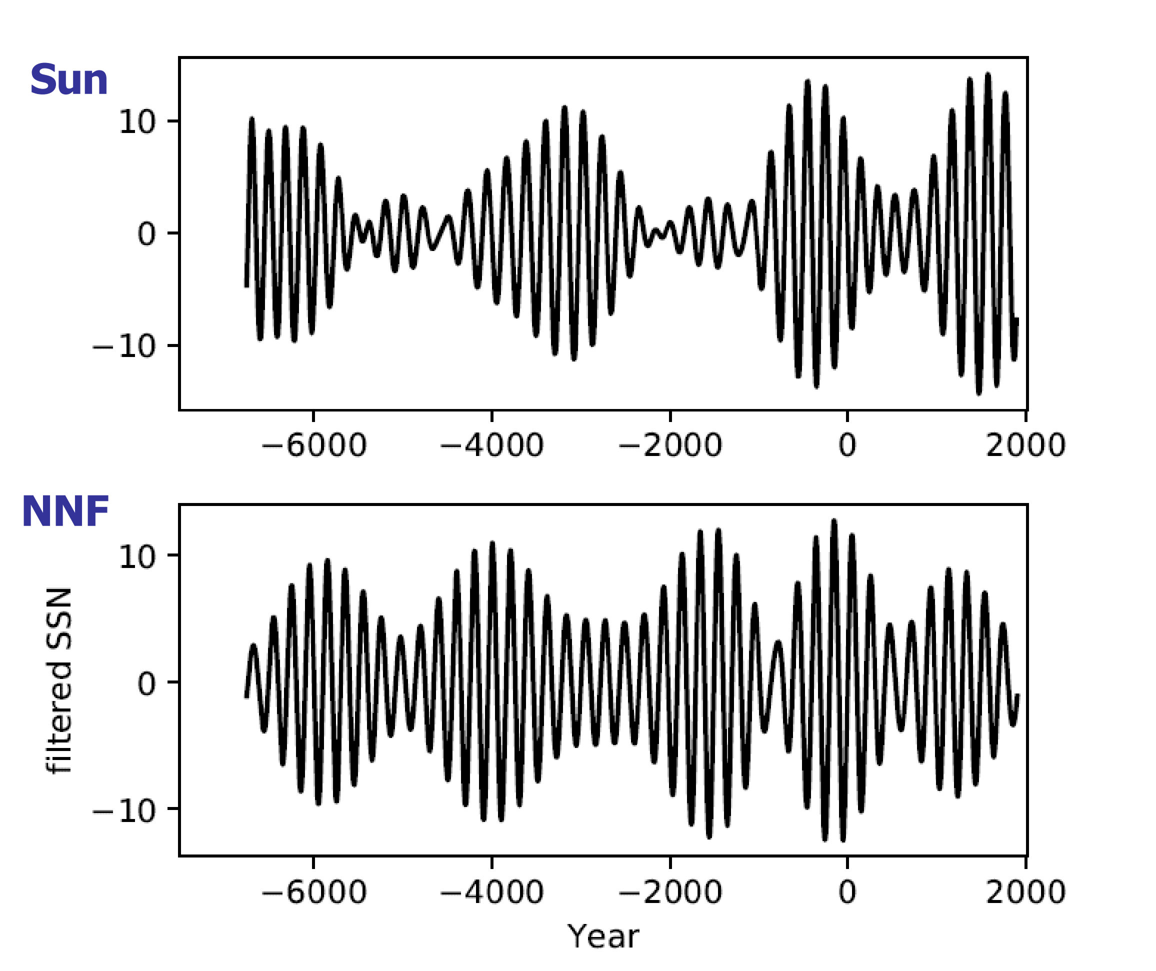
→ estimates require quantitative theoretical understanding of the interaction of convection, rotation, and magnetic field (reliable simulations!)

→ complexity!

Passband-filtered records



Gleissberg domain: 75 yr – 100 yr



De-Vries domain: 180 yr – 230 yr

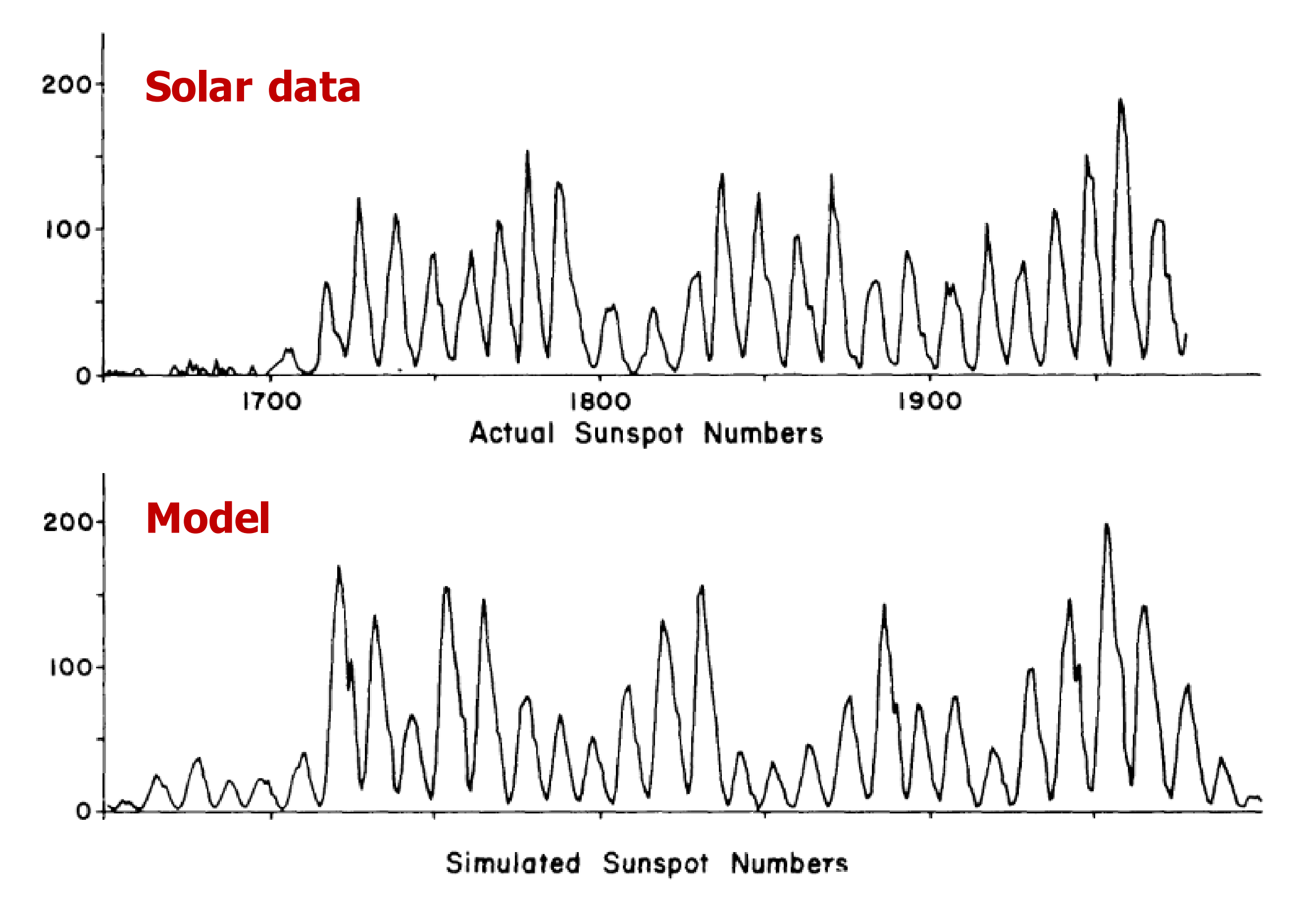
The simplest solar cycle model ever…

Sunspot cycle simulation using random noise

J. A. Barnes[1], H.H. Sargent III[2], and P. V. Tryon[1]

[1]National Bureau of Standards, Boulder, Colorado 80303 and [2]National Atmospheric and Oceanic Administration, Boulder, Colorado 80303

„The ancient Sun” (eds. Pepin, Eddy & Merrill; 1980)

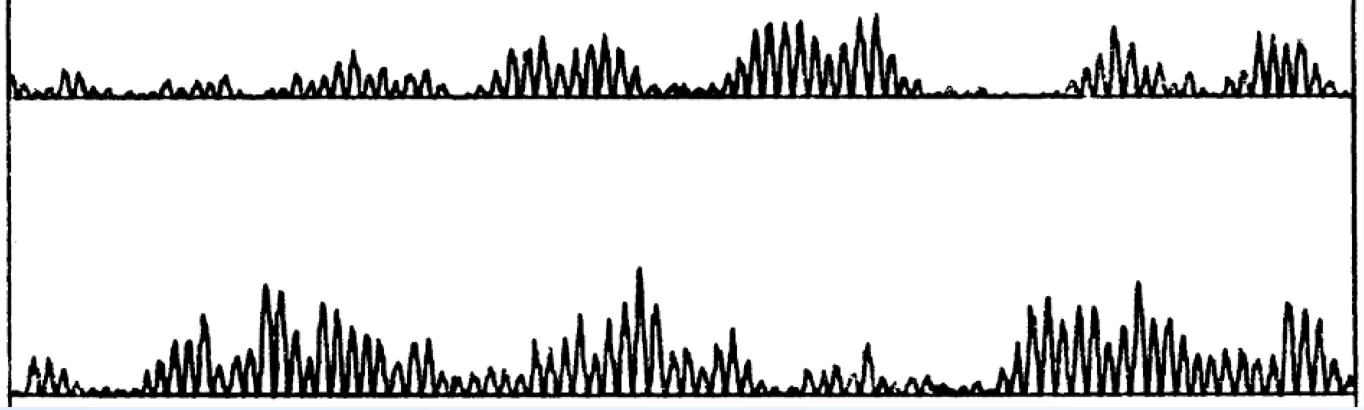


The simplest solar cycle model ever…

Barnes et al. (1980): Auto-regressive moving-average (ARMA) model (iterative map)
→ white noise filtered around 1/22 cyc/year with bandwith 0.002 cyc/year

Long-term evolution:

Model results covering 2000 years…



The full code:

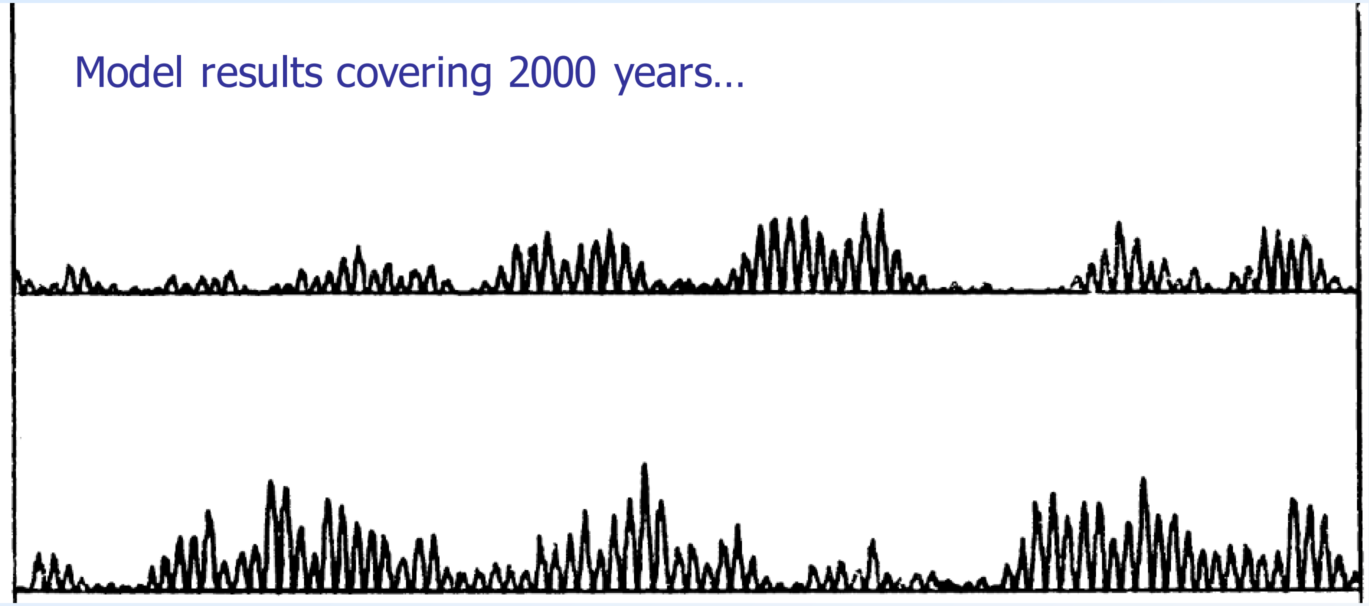
```
100 X = RND (−2) : A = 1.90693 : B = − .98751
110 C = .78512 : D = − .40662 : E = .4 : F = .03 : G = 0
130 FOR N = 1 TO 300
140 IF G = 1 THEN GOTO 180
150 X = RND (X) : Y = SQR ( − 2 * LOG (X))
160 X = RND (X) : K = Y * E * COS (6.28318 * X)
170 G = 1 : GOTO 190
180 G = 0 : K = Y * E * SIN (6.28318 * X)
190 H = A * I + B * J + K − C * L − D * M
200 M = L : L = K : S = I * I − J * J
210 T = H * H + F * S * S
220 PRINT T
230 J = I : I = H
240 NEXT N
250 STOP
```

appropriately written in **BASIC:**
Beginner's **A**ll-purpose **S**ymbolic **I**nstruction **C**ode

The simplest solar cycle model ever…

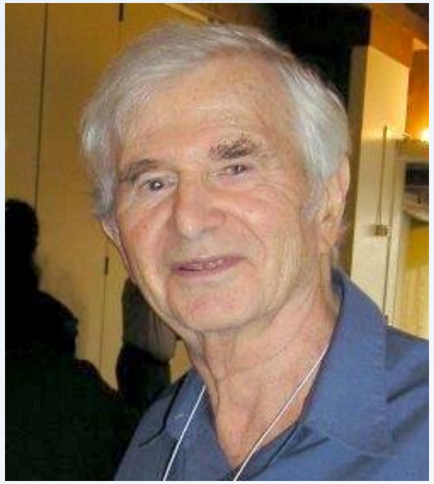
Barnes et al. (1980): Auto-regressive moving-average (ARMA) model (iterative map)
$\rightarrow$ white noise filtered around 1/22 cyc/year with bandwith 0.002 cyc/year

Long-term evolution:



Is there anything we could learn from such a 'model'?

" The purpose of models is not to fit the data but to sharpen the questions."



Samuel Carlin

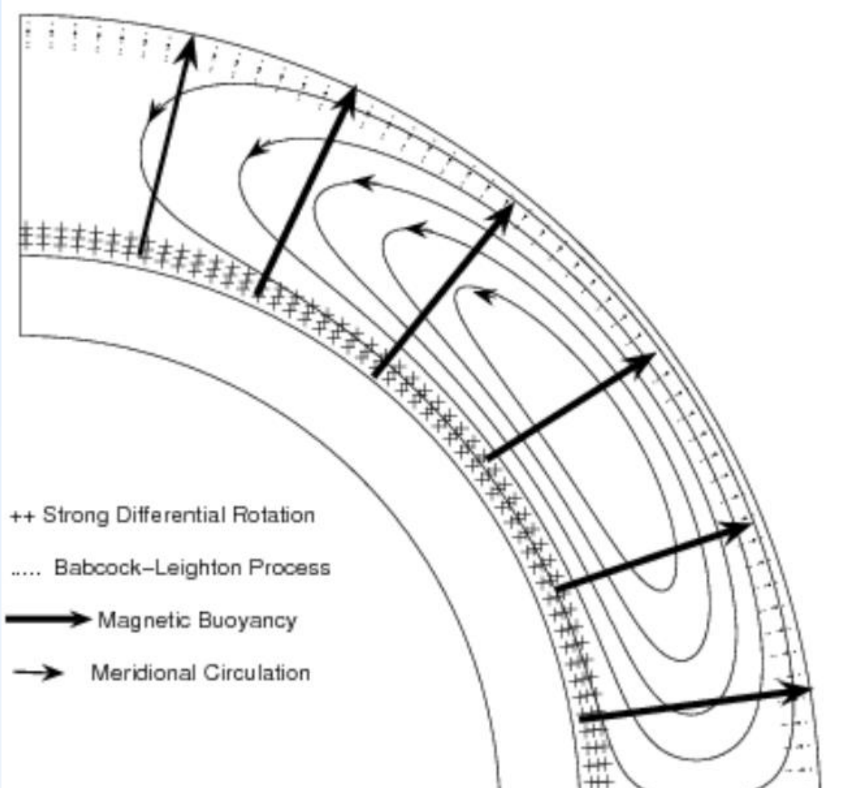
Does randomness cause the variability of the solar cycle?

What is the „current paradigm" ?

Poleward and downward transport of poloidal field by meridional circulation and/or turbulent diffusion and/or pumping ✓?

Toroidal field mainly generated by radial differential rotation in the tachocline ?

Toroidal field stored in a stable layer and transported equatorward by meridional circulation. ?



(Karak et al., 2014)

Poloidal field generated by a near-surface Babcock-Leighton process ✓

Flux tubes destabilize and rise buoyantly, are affected by the Coriolis force, and emerge. ?

Babcock-Leighton 2.0 (Cameron & S., 2017, A&A)

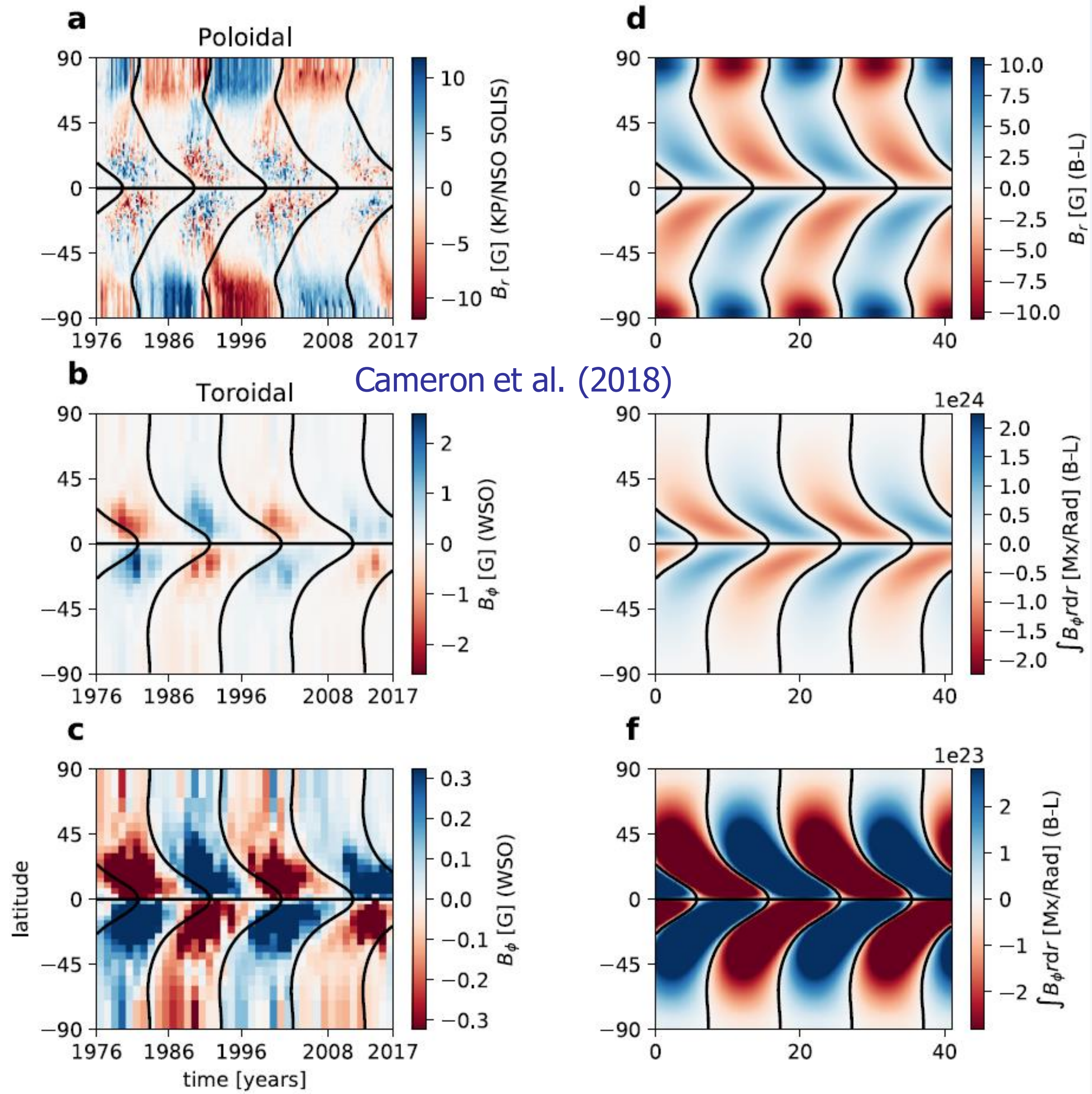
→ **3 parameters, constrained by comparison with observation**

return flow speed: **$V_0 \approx$ 2 … 3 m/s**

turbulent diffusivity: **$\eta_0 \approx$ 30 … 80 km²/s**

source strength: **$\alpha \approx$ 1 … 3 m/s**

- solar-like solutions with reasonable parameter values (Cameron & S., 2017a)
- consistent with observed azimuthal surface field (Cameron et al., 2018)
- consistent with spectrum of long-term activity records (Cameron & S., 2017b)
- frequencies of N-S asymmetry (S. & Cameron, 2018)



Cameron et al. (2018)

The crucial question

Q: What is the relevant poloidal flux for the solar dynamo?

Hale's polarity laws imply that bipolar magnetic regions result from a **large-scale toroidal field of fixed orientation** in each hemisphere during a cycle.

⇒ Need to consider the **net toroidal flux** in a hemisphere, determined from the azimuthally averaged induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{U} \times \mathbf{B} + \langle \mathbf{u} \times \mathbf{b} \rangle - \eta \nabla \times \mathbf{B} \right)$$

B(r,θ): azimuthally averaged magnetic field,
U(r,θ): azimuthally averaged velocity,
u, **b** : fluctuations w.r.t. azimuthal averages,
η : molecular diffusivity

What is the relevant poloidal flux?

Determine toroidal flux in the northern hemisphere by integrating over a meridional surface Σ and applying **Stokes' theorem**:

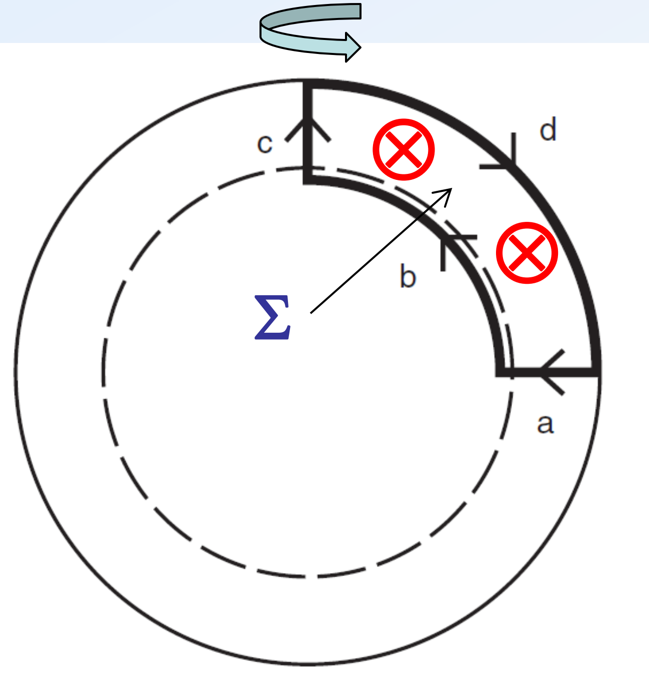
$$\frac{d\Phi_{tor}^{N}}{dt} = \frac{d}{dt}\left(\int_{\Sigma} B_{\phi}\, dS\right)$$

$$= \int_{\delta\Sigma}\Big(\mathbf{U}\times\mathbf{B} + \langle\mathbf{u}\times\mathbf{b}\rangle - \eta\nabla\times\mathbf{B}\Big)\cdot d\mathbf{l}$$

Rotation dominates: $U = U_{\phi}\,\hat{\phi} = (\Omega r \sin\theta)\hat{\phi}$

$\langle\mathbf{u}\times\mathbf{b}\rangle$ reduces to „turbulent" diffusivity, η_t

$$\frac{d\Phi_{tor}^{N}}{dt} = \int_{\delta\Sigma}(\mathbf{U}\times\mathbf{B} - \eta_t\nabla\times\mathbf{B})\cdot d\mathbf{l}$$



Meridional cut

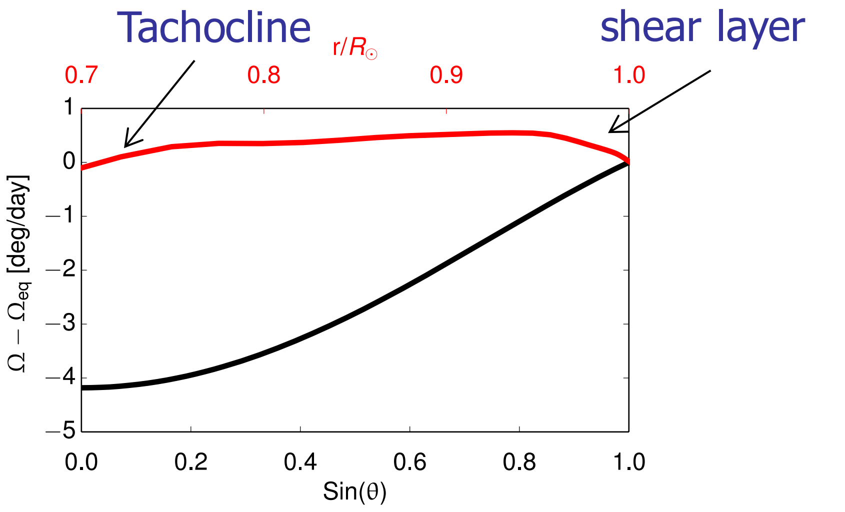
Cameron & S. (2015)

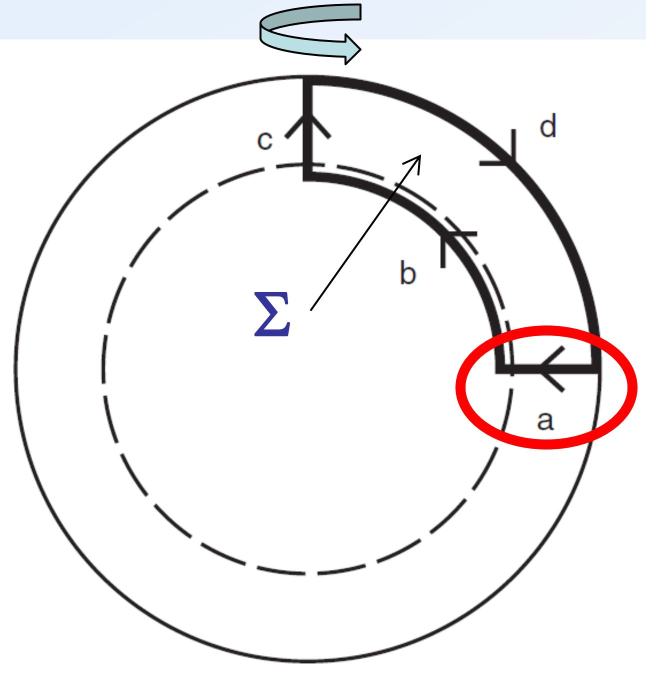
What is the relevant poloidal flux?

Consider $\int_{\delta\Sigma} (\mathbf{U} \times \mathbf{B}) \cdot \mathbf{dl}$

Part a: Ω almost independent of r in the equatorial plane: $\Omega(r, \pi/2) = \Omega_{\rm eq}$

Move in a frame rotating with $\Omega_{\rm eq}$
$\rightarrow$ no contribution





Meridional cut

Cameron & S. (2015)

What is the relevant poloidal flux?

Consider $\int_{\delta\Sigma} (\mathbf{U} \times \mathbf{B}) \cdot d\mathbf{l}$

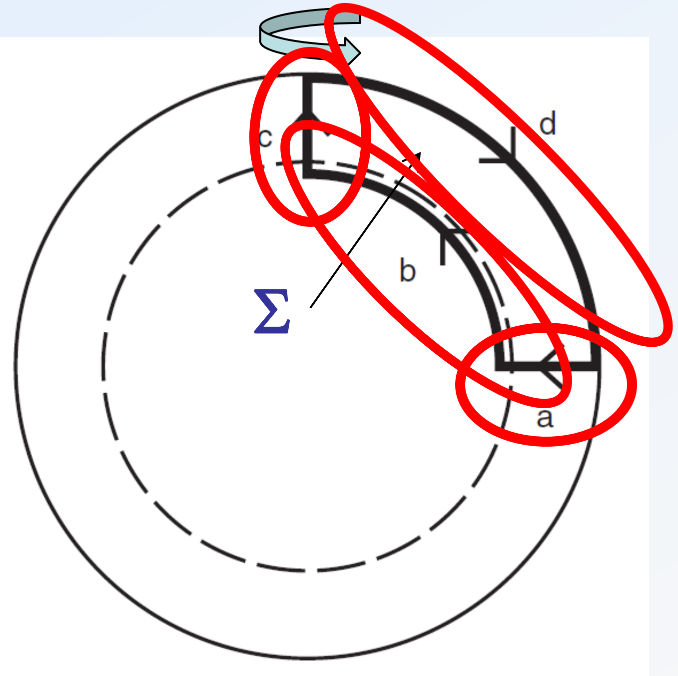
Part a: Ω almost independent of r in the equatorial plane: $\Omega(r, \pi/2) = \Omega_{\text{eq}}$

Move in a frame rotating with Ω_{eq}
$\rightarrow$ no contribution

Part b: below convection zone, B=0
$\rightarrow$ no contribution

Part c: along the axis, B=U=0
$\rightarrow$ no contribution

Part d: the surface part of the integration provides the only significant contribution



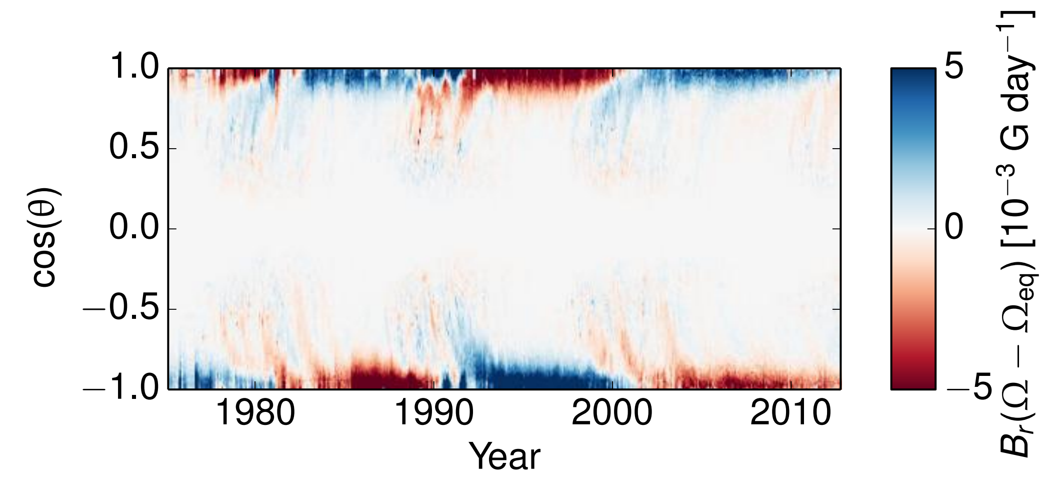
Meridional cut

Cameron & S. (2015)

What is the relevant poloidal flux?

Quantitative evaluation: use Kitt Peak synoptic magnetograms (1975-) and the observed surface differential rotation

$$\int_{\delta\Sigma} (\mathbf{U} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l} = \int_{\pi/2}^{0} U_\phi B_r R_\odot \mathrm{d}\theta = \int_{1}^{0} (\Omega - \Omega_{\rm eq}) B_r R_\odot^2 \, \mathrm{d}(\cos\theta) \, ,$$



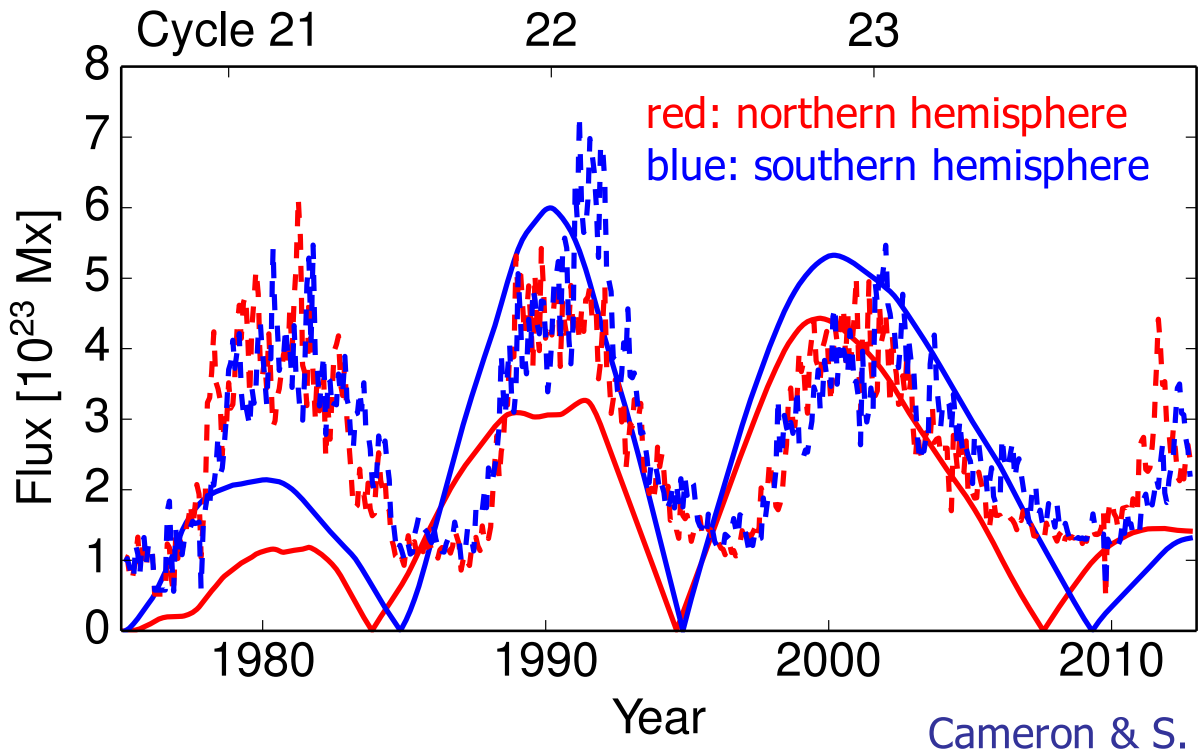
$(\Omega - \Omega_{\rm eq})B_r.$

The integrand is dominated by the contribution from the polar fields.

Cameron & S. (2015)

What is the relevant poloidal flux?

Time integration of $$\frac{d\Phi^{N}_{tor}}{dt} = \int_0^1 (\Omega - \Omega_{eq}) B_r R_\odot^2 \, d(\cos\theta)$$

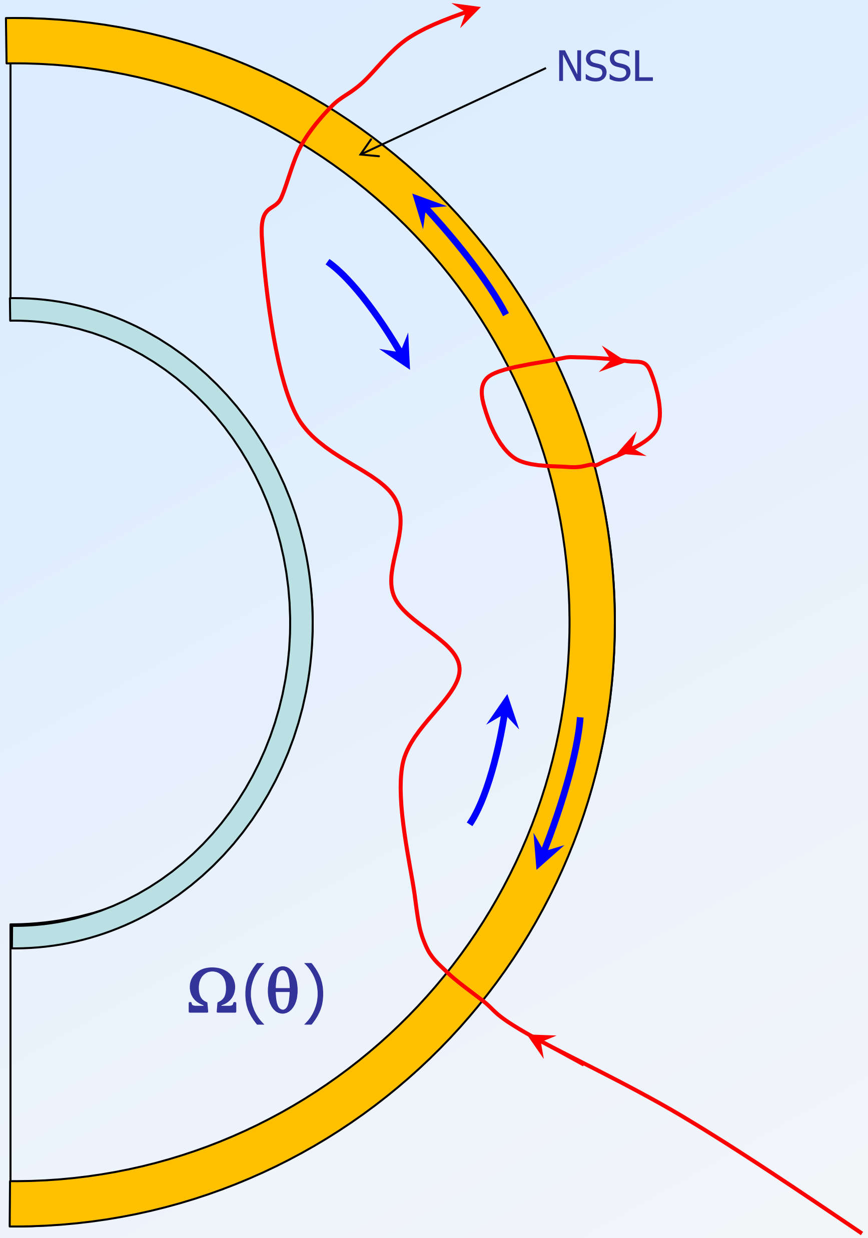


Cameron & S. (2015)

solid: modulus of the net toroidal flux

dashed: total unsigned surface flux, $\Phi_U = \int |B_r| dA$ (KPNO synoptic magnetograms)

Babcock-Leighton 2.0



An update of the model (Cameron & S., 2017, A&A) takes into account information not available to B&L:

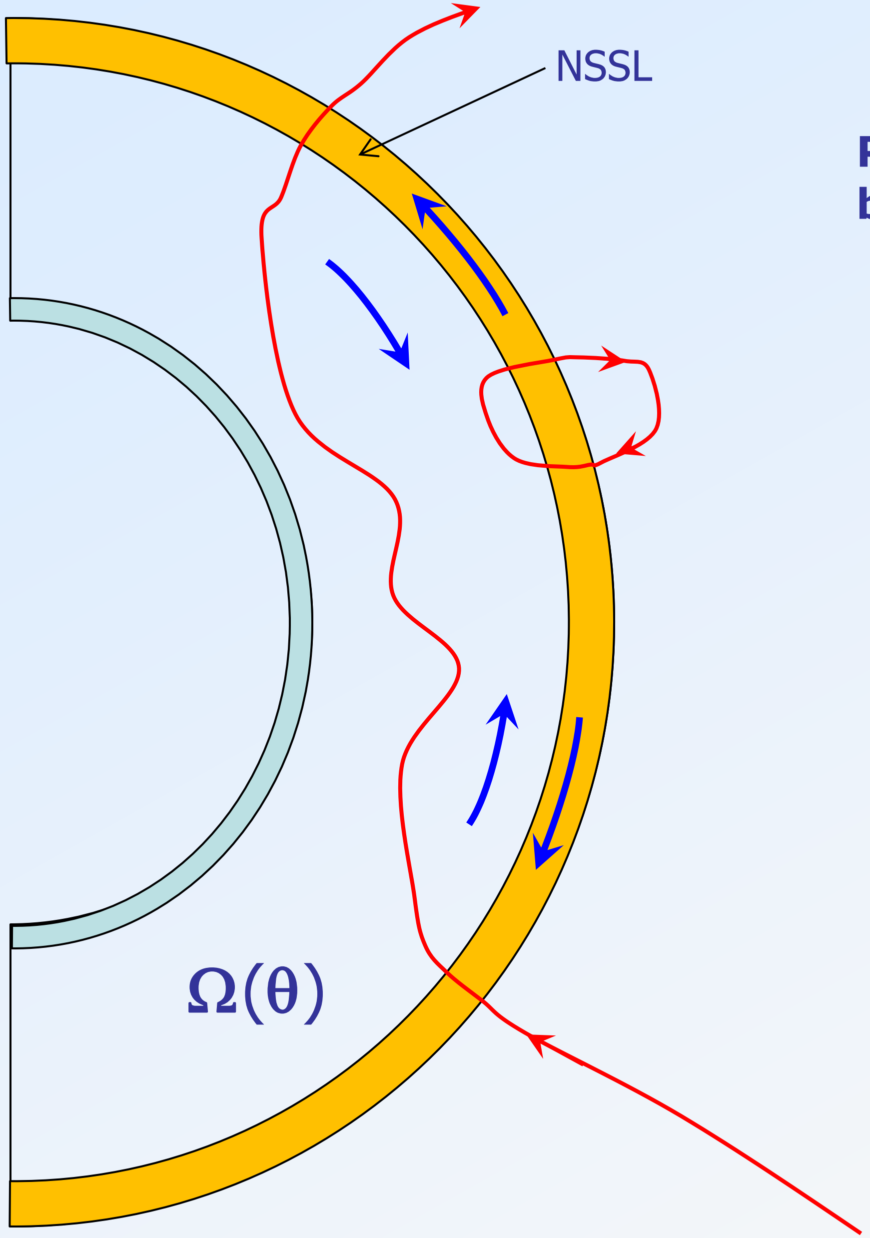
→ differential rotation in the convection zone
→ near-surface shear layer
→ meridional flow
→ (turbulent) magnetic diffusivity affecting Btor
→ convective pumping
→ randomness in flux emergence

Consider radially integrated toroidal flux
and radial surface surface field

⇨ parameter space significantly reduced to basically three parameters:

→ turbulent diffusivity
→ poloidal source strength
→ speed of meridional return flow

Babcock-Leighton 2.0



Parameter values strongly constrained by observation:

- Dynamo period: ~22 years
- Phase difference between maxima of flux emergence (activity) and polar fields : ~90 deg
- Weak excitation: dipole mode excited,
quadrupole mode decaying

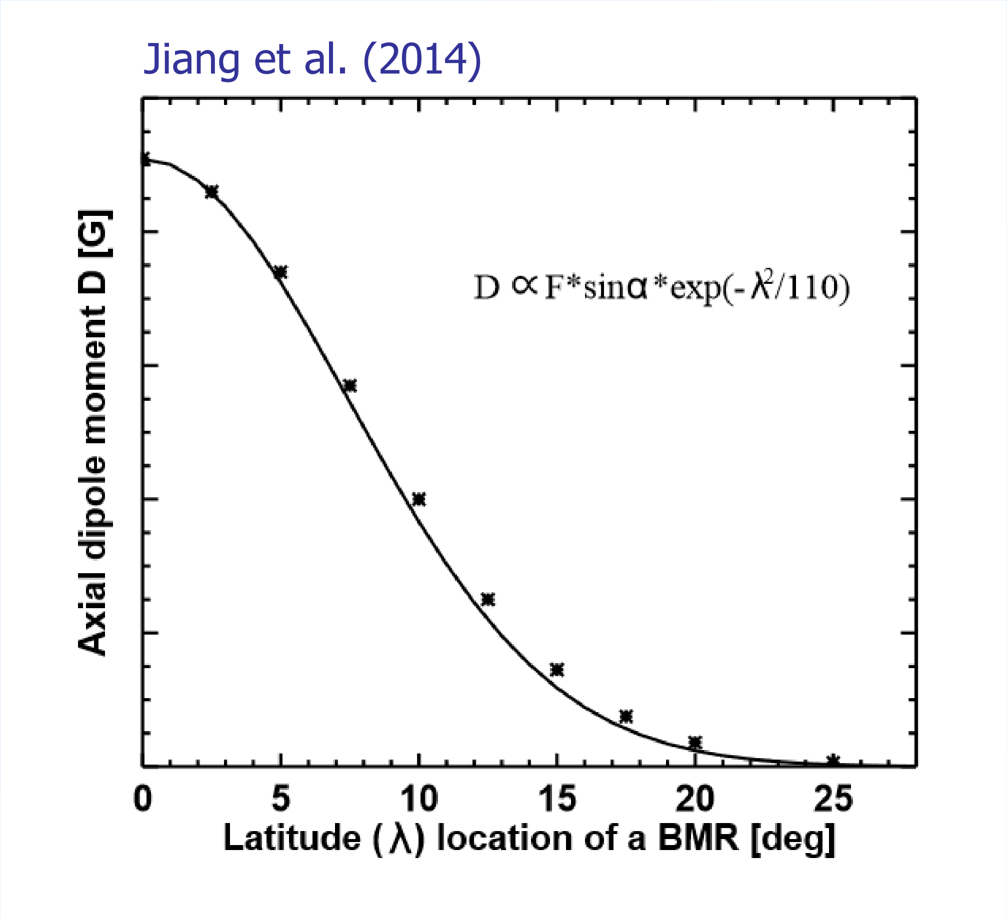
→ Constraints:

return flow speed: $V_0 \approx$ **2 … 3 m/s**

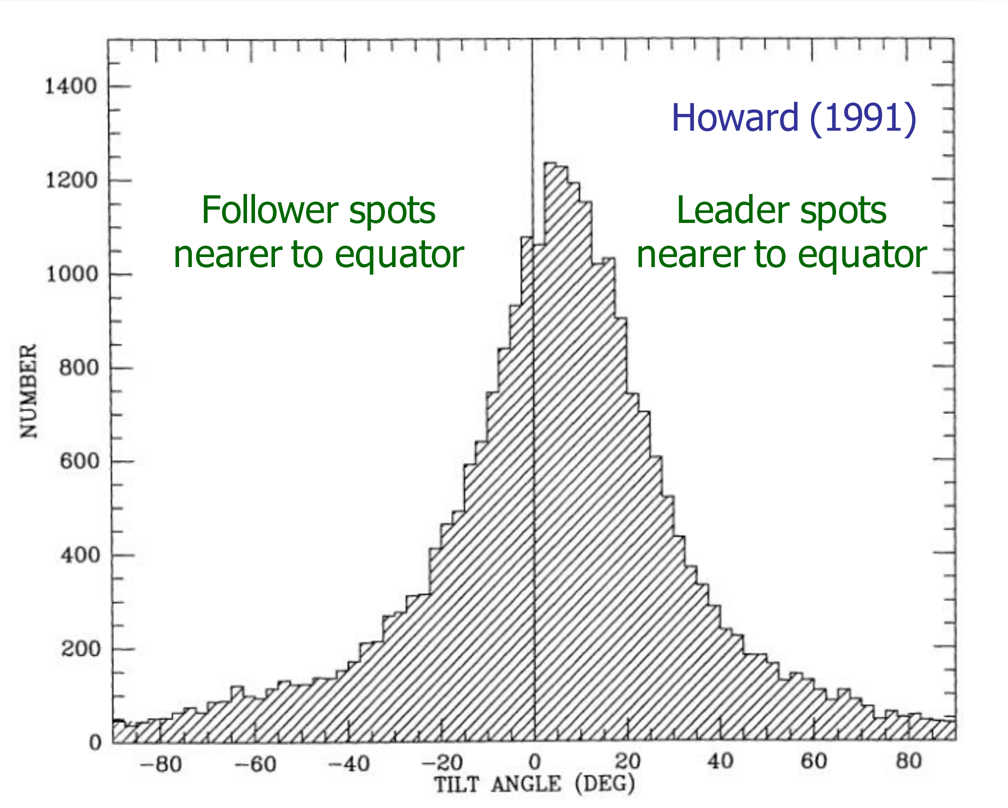
turbulent diffusivity: $\eta_0 \approx$ **30 … 80 km²/s**

source strength: $\alpha \approx$ **1 … 3 m/s**

Randomness matters



Contribution of bipolar magnetic regions with a flux of 6×10^{21} Mx to the axial dipole moment around solar minimum as a function of emergence latitude



Histogram of sunspot group tilt angles
(Mt. Wilson, 1917 – 1985)

The dipole moment around solar minimum – and thus the strength of the next activity cycle – is most strongly affected by the relatively small number of near-equator bipolar magnetic regions.

Substantial **scatter** of sunspot group tilt angles

Key observations and development of dynamo models

before 1960

- 11-year cycle
- surface differential rotation
- equatorward migration of the activity belts
- polarity rules & tilt angles of sunspot groups
- global dipole field & reversals

→ Parker loop (1955), Babcock scenario (1961), Leighton model (1964/1969),
Mean-field electrodynamics & „turbulent dynamos" (1960s onward)

1980s…

- poleward surface meridional flow
- internal differential rotation, tachocline
- long-term synoptic maps of the surface field

→ Surface flux transport simulations (Wang & Sheeley, …)
Flux transport dynamo models, Babcock-Leighton revival

1990s…today

- time-dependent deep zonal flows
- flows associated with active regions (e.g., near-surface inflows)
- flows connected to flux emergence
- deep meridional flow

→ Spherical 3D MHD comprehensive simulations

The 1990s and beyond: new aspects

- dynamo effect of magnetic instabilities („dynamic dynamo”)
- fast and slow dynamos (growth rate finite as $R_m \rightarrow \infty$?)
- conservation of magnetic helicity
- stochastic fluctuations of the dynamo coefficients
- nonlinear dynamics, chaos and intermittency $\rightarrow$ grand minima ?
- (partial) recovery of mean-field models:
  consistent combination of the generation of differential rotation („Λ-effect”) and magnetic field (Kitchatinov, Rüdiger, et al.)
- idealized box simulations show dynamo action
  for helical/non-helical as well as turbulent/laminar flows
- small-scale dynamo action at low magnetic Prandtl number, R_m/Re ?
- direct simulations in spherical shells (Brun et al.) greatly improved,
  but still no solar-like large-scale fields
  (compare with the success of realistic simulations of surface magneto-convection)

Magnetic buoyancy

$$p_i = p_e - \frac{B^2}{8\pi} \quad \& \quad T_i = T_e \quad \Rightarrow \quad \rho_i < \rho_e$$

$\rightarrow$ (magnetic) buoyancy

Example: Horizontal flux tube

Density difference : $\dfrac{\Delta\rho}{\rho_i} = \dfrac{(\rho_i - \rho_e)}{\rho_i} = -\dfrac{B^2}{8\pi p_i} = -\dfrac{1}{\beta}$

Buoyancy force : $F_B = -\Delta\rho\, g = \dfrac{B^2}{8\pi H_p}$

$(T_i = T_e; H_p = \mathcal{R}T/\mu g \simeq 6 \cdot 10^4$ km; pressure scale height$)$

Balance by aerodyn. drag force: $F_D = (C_D/\pi a)\, \rho_e\, v_\perp^2$

$$F_D = F_B \quad \rightarrow \quad v_\perp \simeq v_A \left(\frac{a}{H_p}\right)^{1/2} \simeq 30\,\mathrm{m/s}$$

$v_A = B/\sqrt{4\pi\rho_e} \simeq 100$ m/s (Alfvén velocity)

$a \simeq 6 \cdot 10^3$ km ($\Phi_{\mathrm{mag}} = 10^{22}$ Mx, $B = 10^4$ G)

$\Rightarrow$ Rises through the whole convection zone in 2 months

BL 2.0: Babcock-Leighton updated

[Cameron & S., 2017]

Why update an ancient model in the era of Flux Transport Dynamos & 3D simulations?

- The structure of convection, magnetic field, and meridional circulation in the convection zone is unknown:
  FTD models require extensive (arbitrary) parametrization
  and 3D MHD models probably run in the wrong physical regime
  $\rightarrow$ a fully realistic dynamo model is not possible at the moment

- The BL model captures the essential physical processes and can be based as far as possible on observations.
  Unknown conditions are condensed in a few free parameters.

- Long time series (thousands of cycles) and extended parameter studies can be carried out easily.

BL 2.0: Babcock-Leighton updated

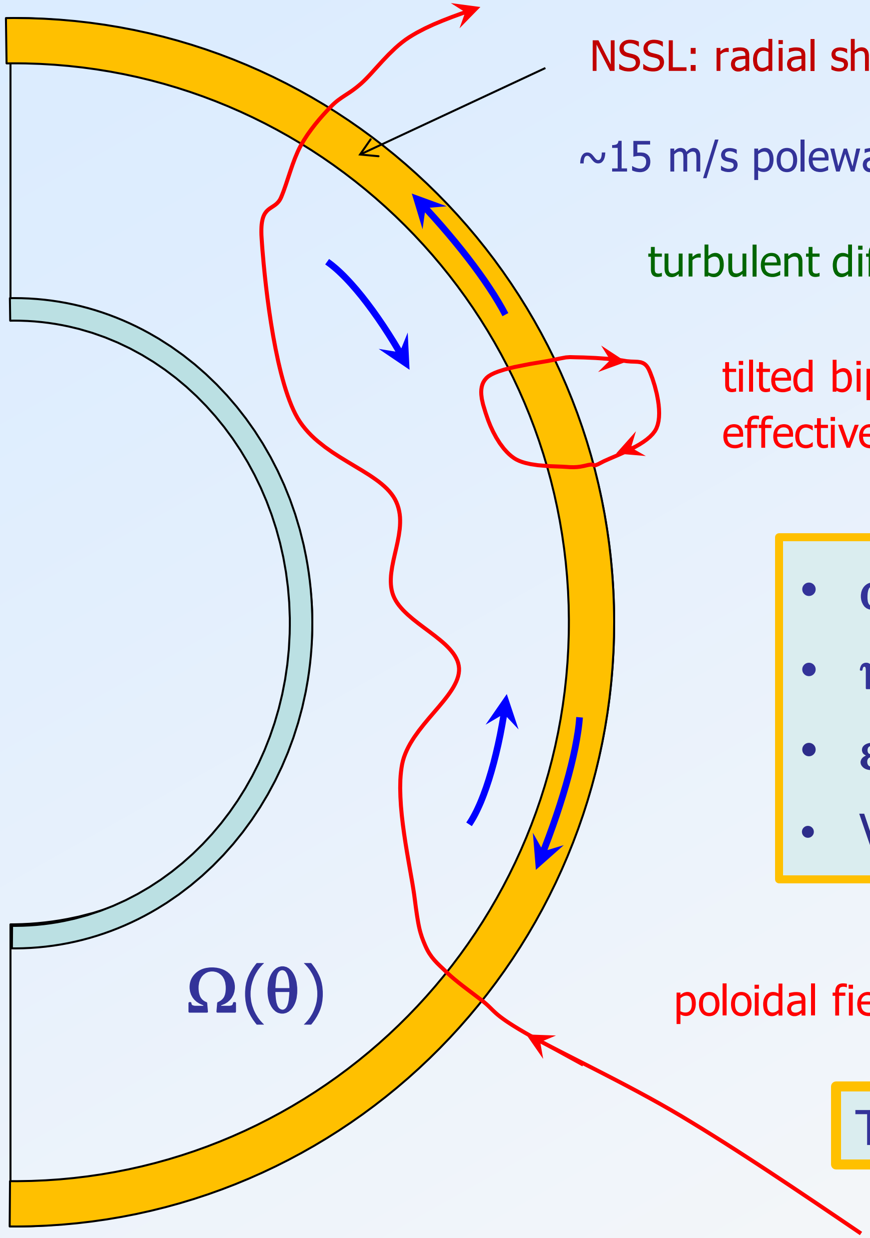
Leighton's model (1969):

- two-layer model: surface $<B_r>$ and radially averaged near-surface $<B_\phi>$
- turbulent diffusion (random walk) of surface field
- latitudinal differential rotation and near-surface shear layer
- flux eruption in tilted bipolar magnetic regions
  serves as nonlinearity and source of poloidal field

Update Leighton's model taking into account:

- surface $<B_r>$ and radially integrated toroidal flux (per unit latitude)
- poleward meridional flow at the surface
- equatorward return flow *somewhere* in the convection zone
- radial differential rotation in the near-surface shear layer (NSSL)
- dominant latitudinal differential rotation below the NSSL
- downward convective pumping of horizontal field in NSSL
- turbulent diffusion also for $<B_\phi>$

Babcock-Leighton 2.0



NSSL: radial shear, radial magnetic field (through pumping)

~15 m/s poleward meridional flow @ surface

turbulent diffusivity @ surface (B_r): 250 km^2/s

tilted bipolar magnetic regions:
effective surface α

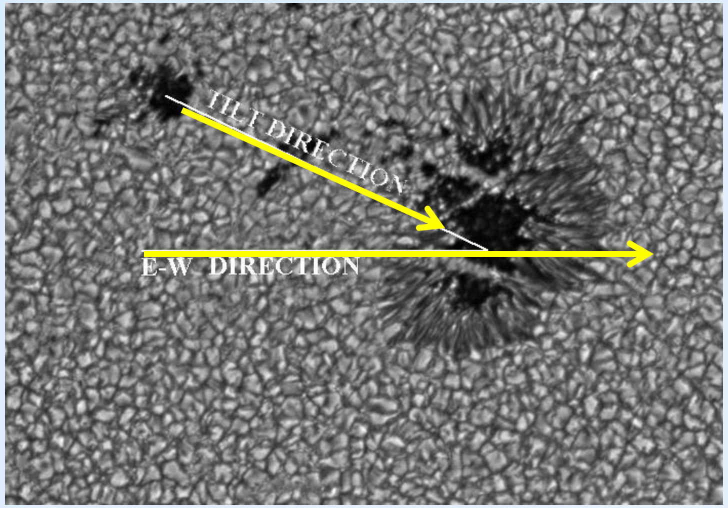
- α strength of the Coriolis effect
- η_{CZ} turbulent diffusivity affecting $b(\theta,t)$
- ε radial shear below NSSL
- V_0 effective merid. return flow affecting $b(\theta,t)$

poloidal field (turns over above tachocline)

Test by comparison with 2D FTD models

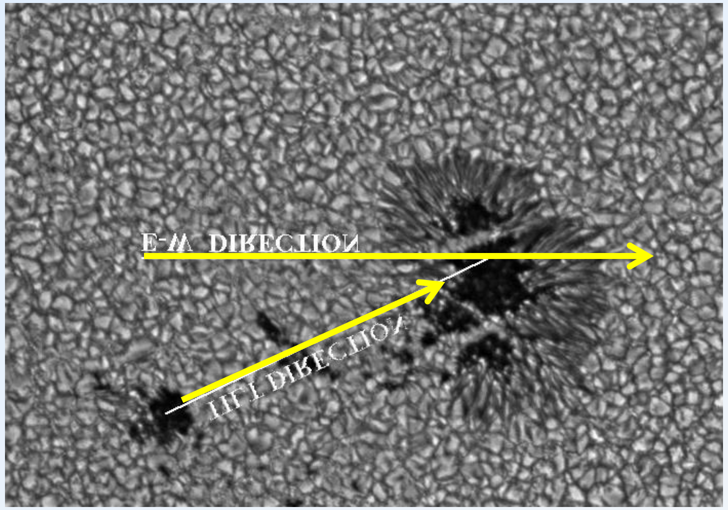
Systematic tilt angle of sunspot groups

Solar rotation



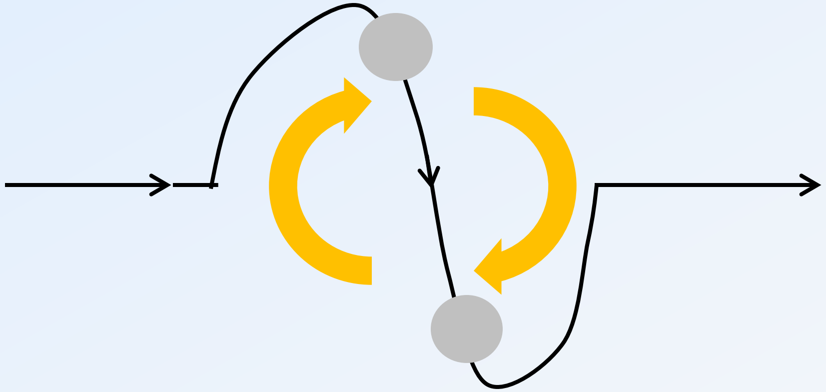
Northern hemisphere

Equator

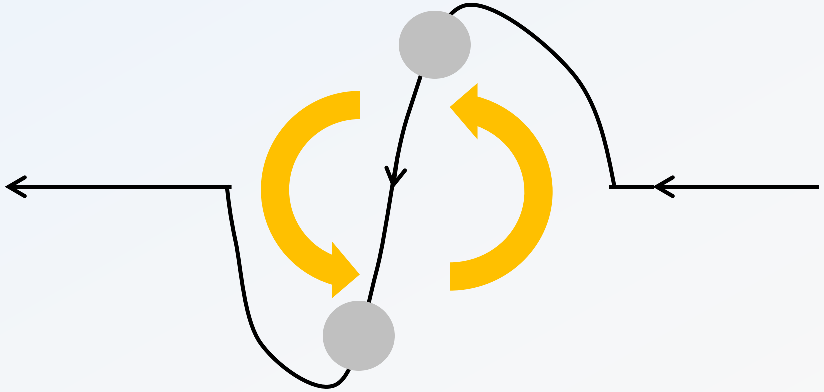


Southern hemisphere

Consistent with the Coriolis effect on rising & expanding loops of magnetic flux



Solar rotation



BL 2.0: toroidal field

$$\frac{\partial B_\phi}{\partial t} = \frac{1}{r}\left\{\frac{\partial\left[r\left(U_\phi B_r - U_r B_\phi\right)\right]}{\partial r} + \frac{\partial\left(U_\phi B_\theta - U_\theta B_\phi\right)}{\partial\theta} + \frac{\partial}{\partial r}\left[\eta\frac{\partial(rB_\phi)}{\partial r}\right] + \frac{\partial}{\partial\theta}\left[\frac{\eta}{r\sin\theta}\frac{\partial(B_\phi\sin\theta)}{\partial\theta}\right]\right\}.$$

$$\frac{\partial B_{r,R_\odot}}{\partial t} = -\frac{1}{R_\odot\sin\theta}\frac{\partial}{\partial\theta}\left(U_{\theta,R_\odot}B_{r,R_\odot}\sin\theta\right) + \frac{\eta_{R_\odot}}{R_\odot^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial B_{r,R_\odot}}{\partial\theta}\right) + S(\theta,t)$$

↓

$$b(\theta,t) = \int_{R_b}^{R_\odot} B_\phi r \mathrm{d}r\,.$$

↓

$$a(\theta,t) = \frac{1}{\sin\theta}\int_0^\theta \sin\theta R_\odot^2 B_{r,R_\odot}\mathrm{d}\theta\,,$$

Differential rotation

$$\frac{\partial b}{\partial t} = \frac{\partial a\sin\theta}{\partial\theta}\epsilon\left(\Omega_{R_\odot} - \Omega_{R_{NSSL}}\right) - \left(\frac{\partial\Omega_{R_{NSSL}}}{\partial\theta}\right)a\sin\theta - \frac{1}{R_\odot}\frac{\partial(V_0\sin(2\theta)b)}{\partial\theta} + \frac{\eta_0}{R_\odot^2}\frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin(\theta)\,b)\right]$$

Meridional return flow

Turbulent diffusion

BL 2.0: poloidal field

$$\frac{\partial B_\phi}{\partial t} = \frac{1}{r}\left\{\frac{\partial\left[r\left(U_\phi B_r - U_r B_\phi\right)\right]}{\partial r} + \frac{\partial\left(U_\phi B_\theta - U_\theta B_\phi\right)}{\partial\theta} + \frac{\partial}{\partial r}\left[\eta\frac{\partial(rB_\phi)}{\partial r}\right] + \frac{\partial}{\partial\theta}\left[\frac{\eta}{r\sin\theta}\frac{\partial(B_\phi\sin\theta)}{\partial\theta}\right]\right\}.$$

$$\frac{\partial B_{r,R_\odot}}{\partial t} = -\frac{1}{R_\odot\sin\theta}\frac{\partial}{\partial\theta}\left(U_{\theta,R_\odot}B_{r,R_\odot}\sin\theta\right) + \frac{\eta_{R_\odot}}{R_\odot^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial B_{r,R_\odot}}{\partial\theta}\right) + S(\theta,t)$$

$\downarrow$

$$b(\theta,t) = \int_{R_b}^{R_\odot} B_\phi r\mathrm{d}r\,.$$

$\downarrow$

$$a(\theta,t) = \frac{1}{\sin\theta}\int_0^\theta \sin\theta\, R_\odot^2 B_{r,R_\odot}\mathrm{d}\theta\,,$$

Meridional flow

$$\boxed{\frac{\partial a}{\partial t} = -\frac{U_0\sin(2\theta)}{R_\odot\sin\theta}\frac{\partial(a\sin\theta)}{\partial\theta} + \frac{\eta_{R_\odot}}{R_\odot^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial(a\sin\theta)}{\partial\theta}\right) + \alpha\cos\theta\sin\theta\, b(\theta,t)}$$

No artificial restriction of flux emergence to low latitudes!

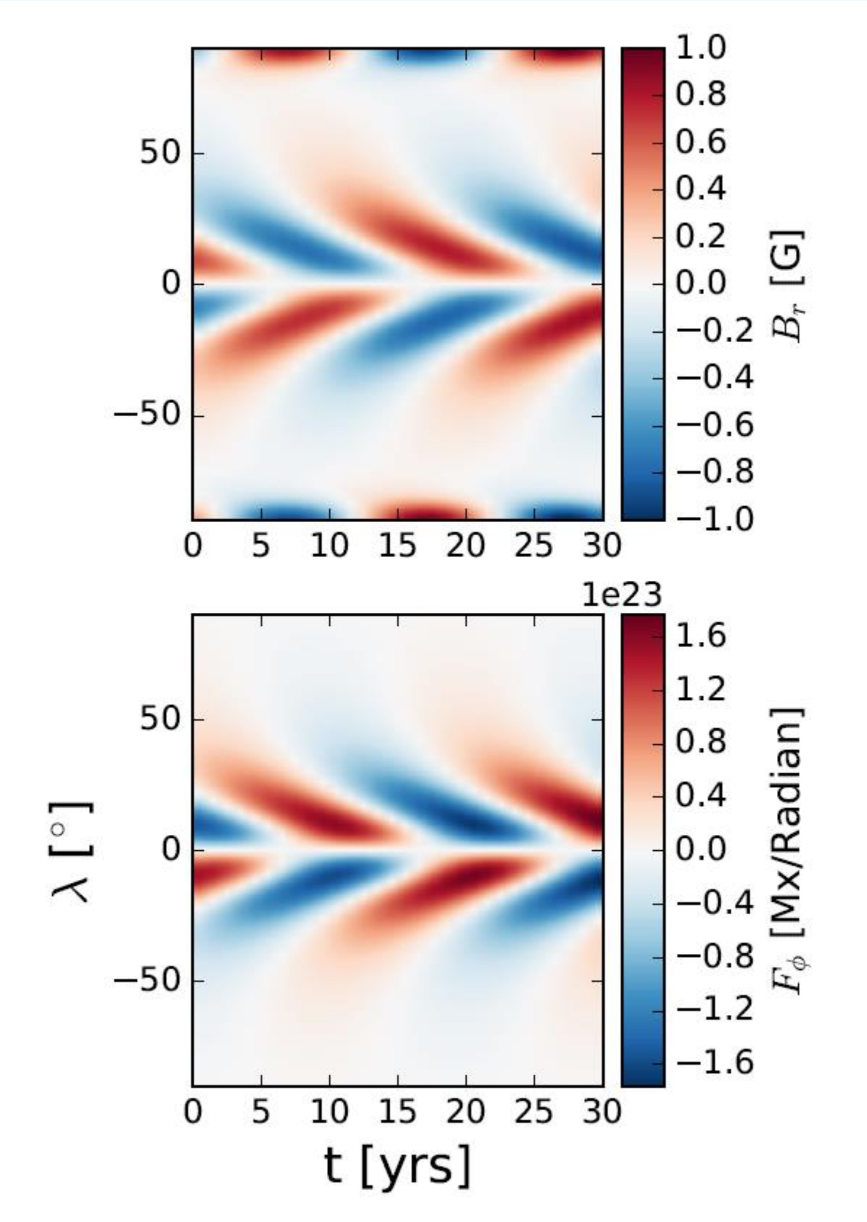
Turbulent diffusion

Tilted bipolar magnetic regions

BL 2.0: example case

Parameters:

η_{CZ} = 80 km^2/s
α = 1.4 m/s
ε = 1.
V_0 = 2.5 m/s

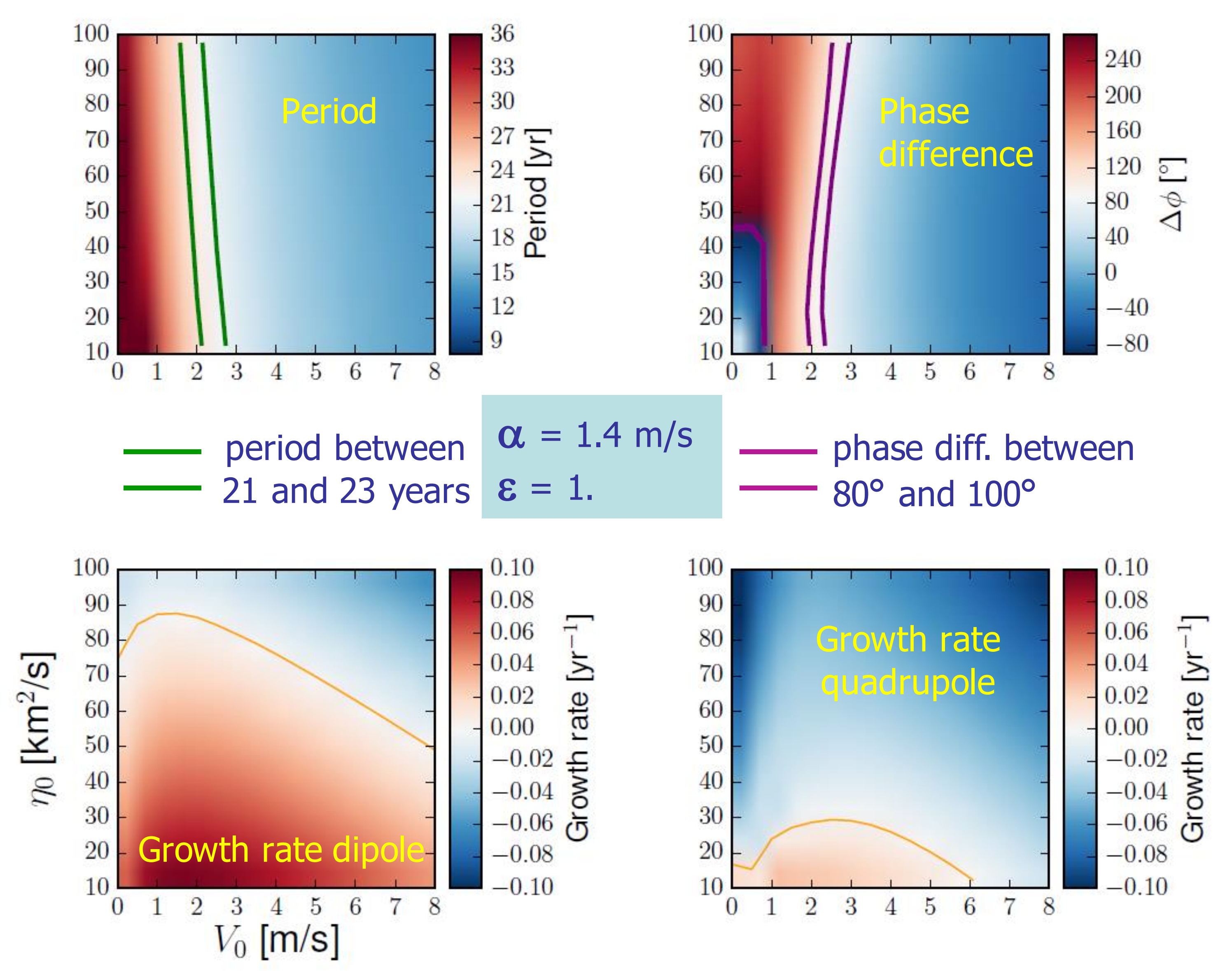


Radial field $B_r(\theta,t)$ @ surface

Toroidal flux $b(\theta,t)$

(Cameron & S., 2017, A&A)

BL 2.0: parameter study



(Cameron & S., 2017, A&A)

Requirements:

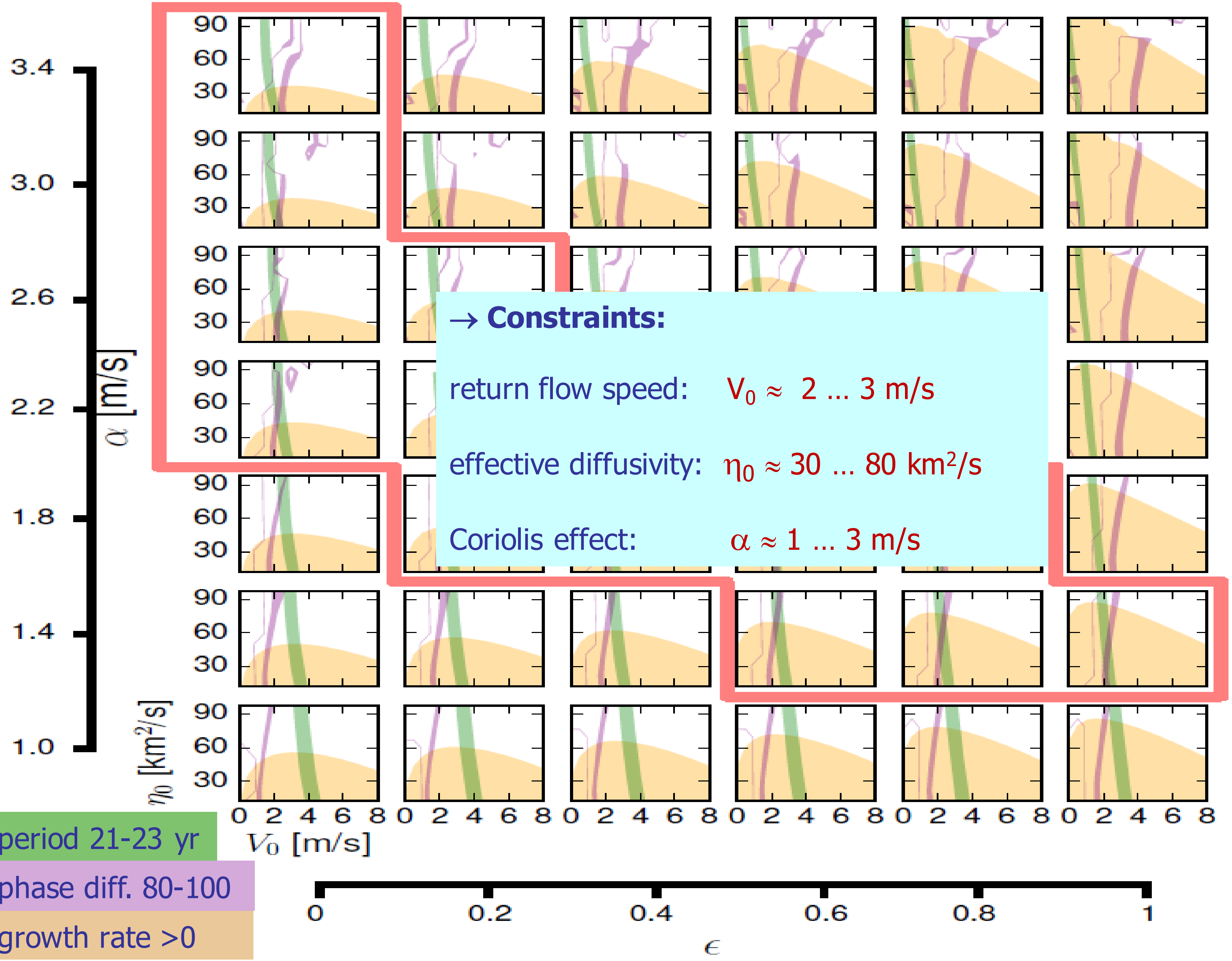
- Period should be ~22 years
- Phase difference between maxima of flux emergence (activity) and polar fields should be ~90 deg
- Dipole mode should be excited, quadrupole mode should be decaying

→ **Constraints:**

return flow speed: $V_0 \approx$ 2 … 3 m/s

effective diffusivity: $\eta_0 \approx$ 30 … 80 km^2/s

Coriolis effect: $\alpha \approx$ 1 … 3 m/s



→ Constraints:

return flow speed: $V_0 \approx$ 2 … 3 m/s

effective diffusivity: $\eta_0 \approx$ 30 … 80 km²/s

Coriolis effect: $\alpha \approx$ 1 … 3 m/s

long-term

Key points in favor of the BL model

- Polar fields reversed and built-up by surface transport of emerged flux
  (flux transport models by Wang & Sheeley + many others)

- Strength of a cycle correlates with the amplitude of the polar fields in the preceding minimum
  (precursor methods for cycle prediction)

- Only flux connected to the surface provides a source for net toroidal flux in a hemisphere. The winding up of the flux connected to the polar fields by (azimuthal) differential rotation generates sufficient toroidal field to cover the flux emerging in the subsequent cycle
  (Cameron & S., 2015)

- BL models with source fluctuations reproduce long-term statistics of activity levels, including grand minima and maxima
  (Cameron & S. 2017, 2019)

- The observed azimuthal surface field (a proxy for flux emergence) evolves in accordance with the updated BL model
  (Cameron et al. 2018)

- The hemispheric asymmetry of solar activity can be quantitatively understood by a superposition of an excited dipole mode and a damped quadrupole mode of the BL dynamo
  (S. & Cameron, 2018)

Key questions and loose ends (general)

- What is the spatial structure and time dependence of the meridional flow?
- Which are the characteristics of deep large-scale convection?
- How is magnetic flux distributed in the convection zone?
- How is flux emergence connected to the structure and distribution of the magnetic field?

⇒ **Helioseismology**, surface observations, comprehensive simulations

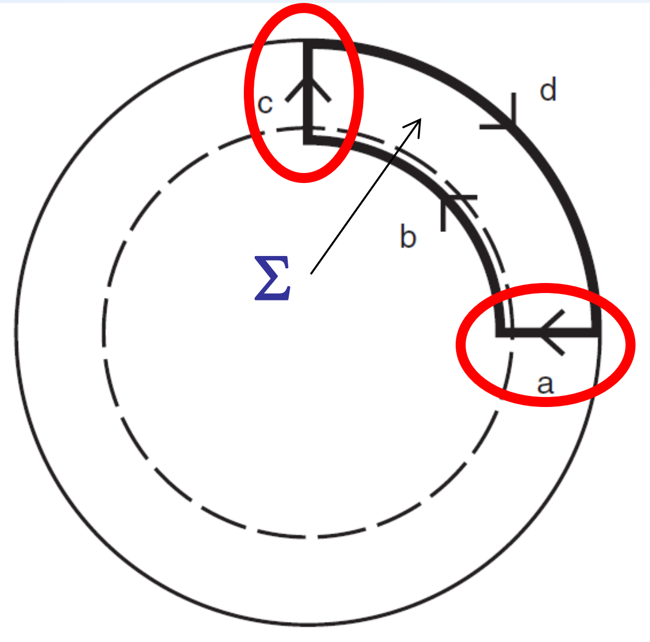
- How can transport of magnetic flux reliably and quantitatively be described in terms of „turbulent diffusion", „turbulent/convective pumping", … ?
- How important are small-scale induction processes within the convection zone (Parker loop, α-effect, …) in comparison to the large-scale Babcock-Leighton mechanism (active region tilt)?

⇒ **Comprehensive simulations**, surface observations

Random walk & „turbulent" diffusivity

Generally we have

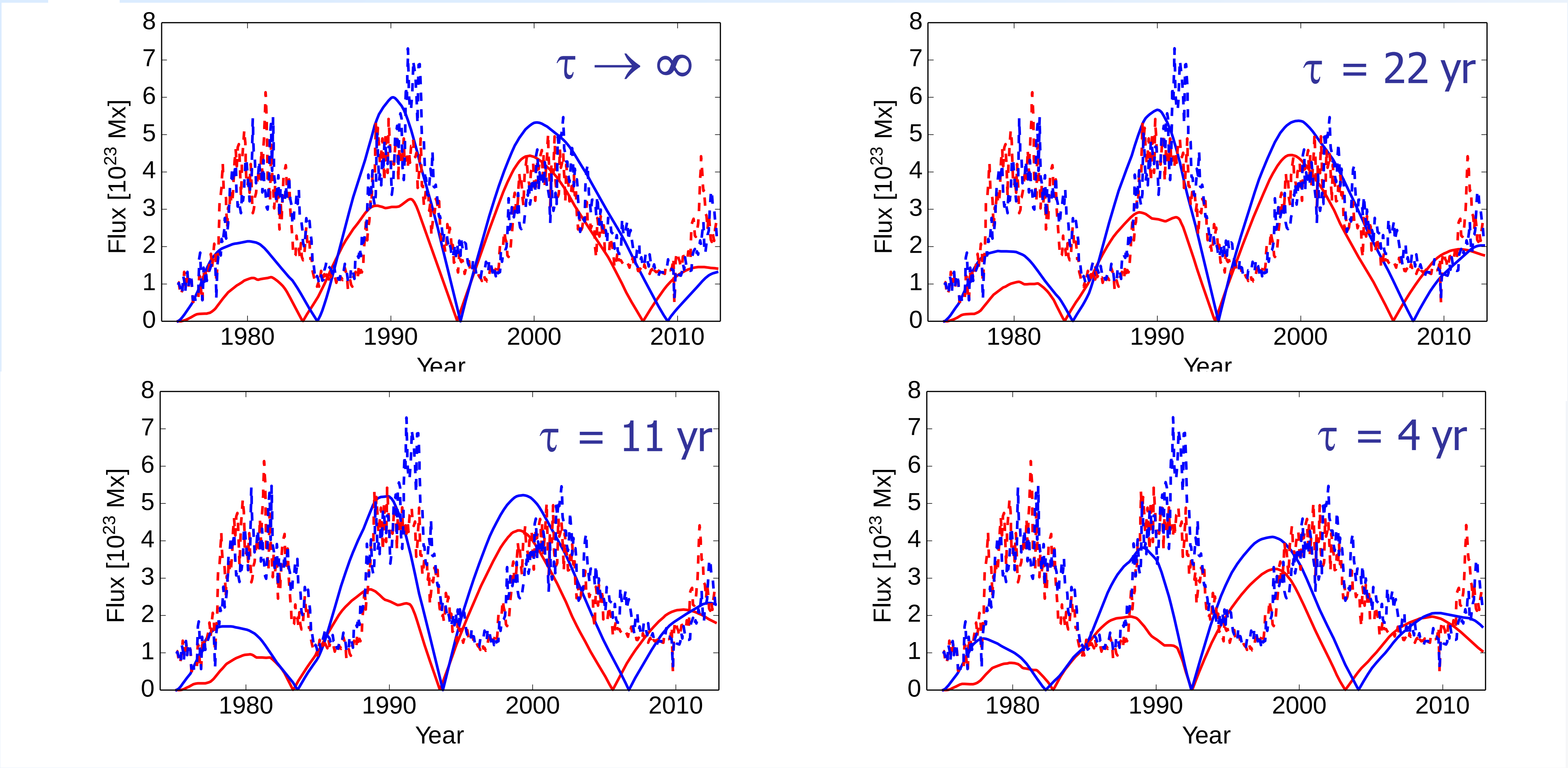
$$\frac{d\Phi^{N}_{tor}}{dt} = \int_{\delta\Sigma} (\mathbf{U} \times \mathbf{B} - \eta_t \nabla \times \mathbf{B}) \cdot d\mathbf{l}$$



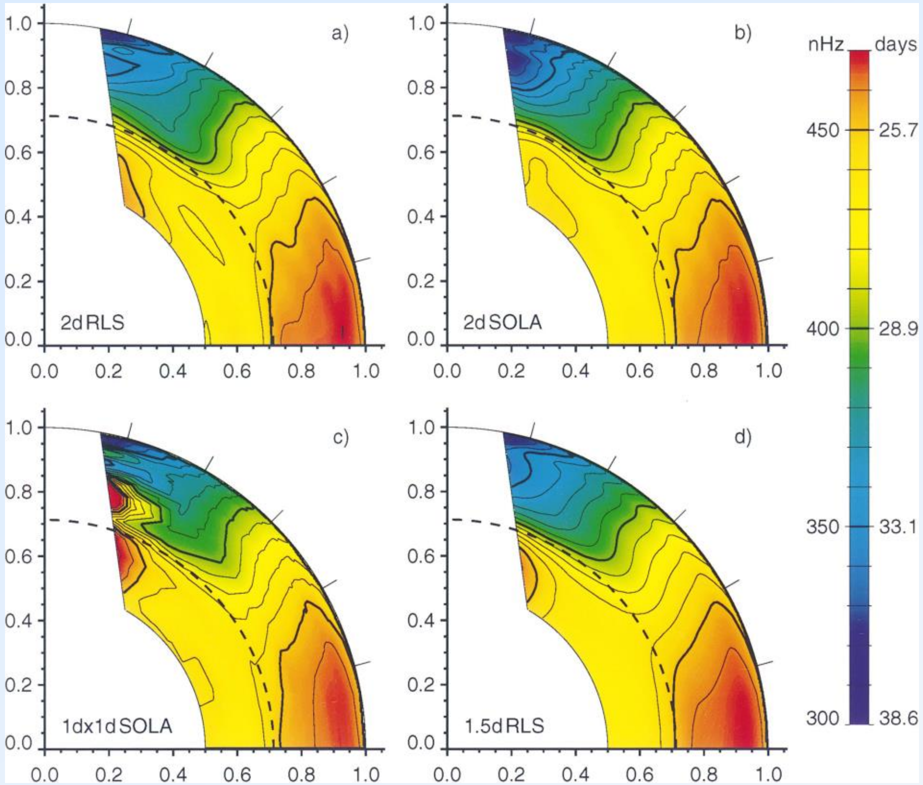
„Turbulent" diffusion (flux loss at the axis and random-walk transport over the equator) is crudely approximated by an exponential decay term:

$$\frac{d\Phi^{N}_{tor}}{dt} = \int_0^1 (\Omega - \Omega_{eq}) B_r R_\odot^2 \, d(\cos\theta) - \frac{\Phi^{N}_{tor}}{\tau}$$

Effect of the decay term



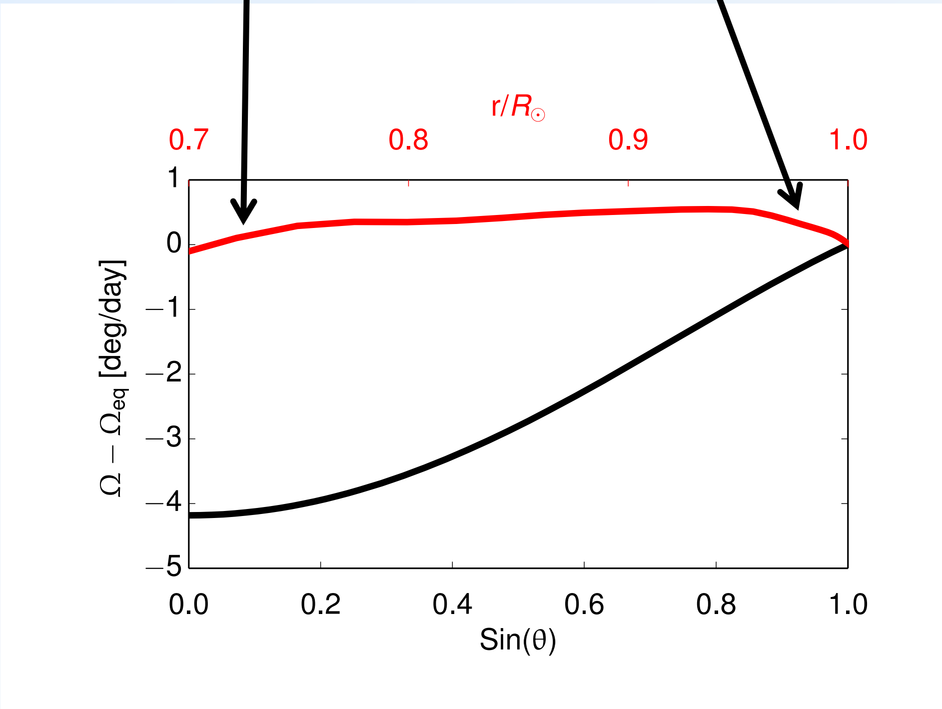
Helioseismology



Schou et al. (1998)

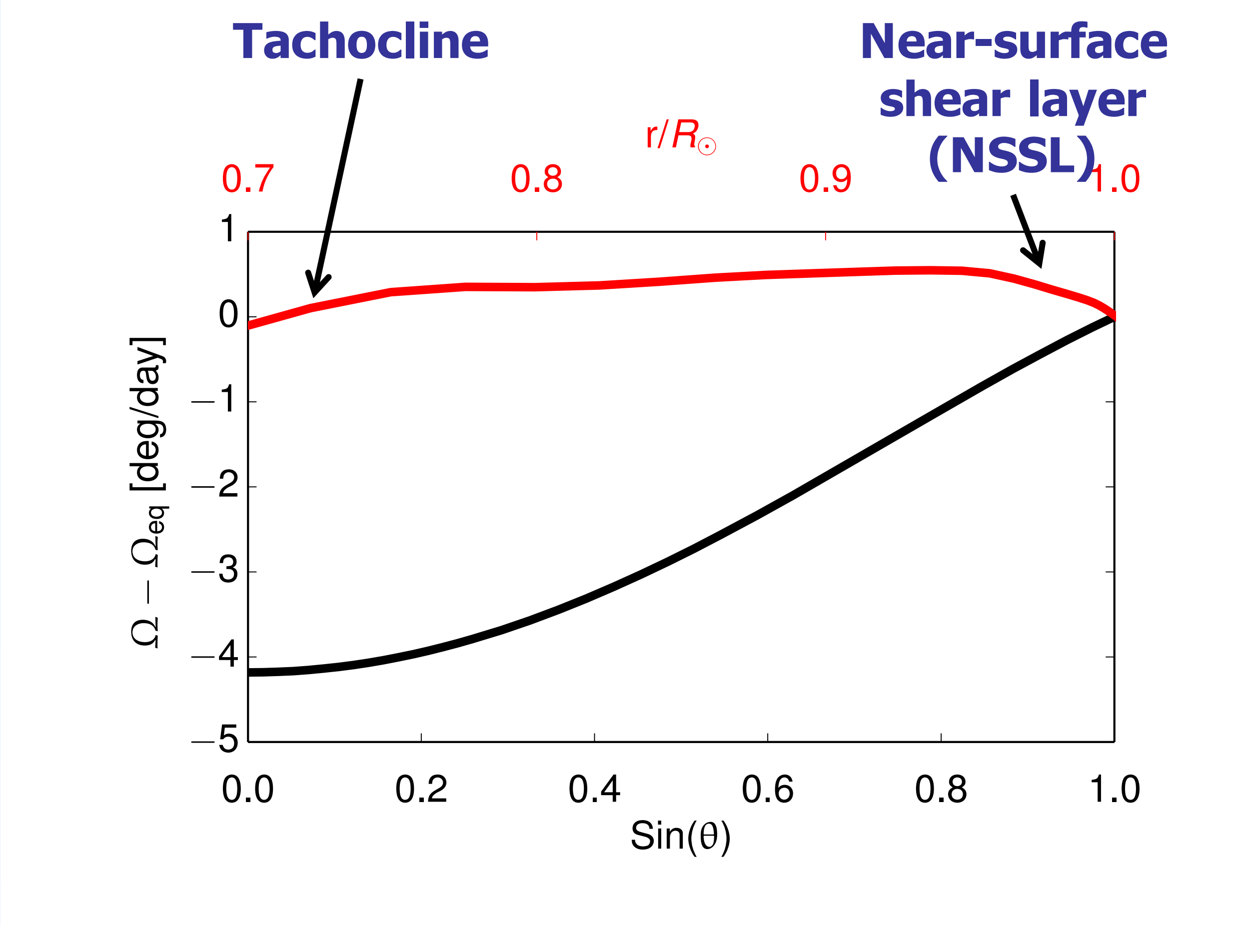
Tachocline

Near-surface shear layer

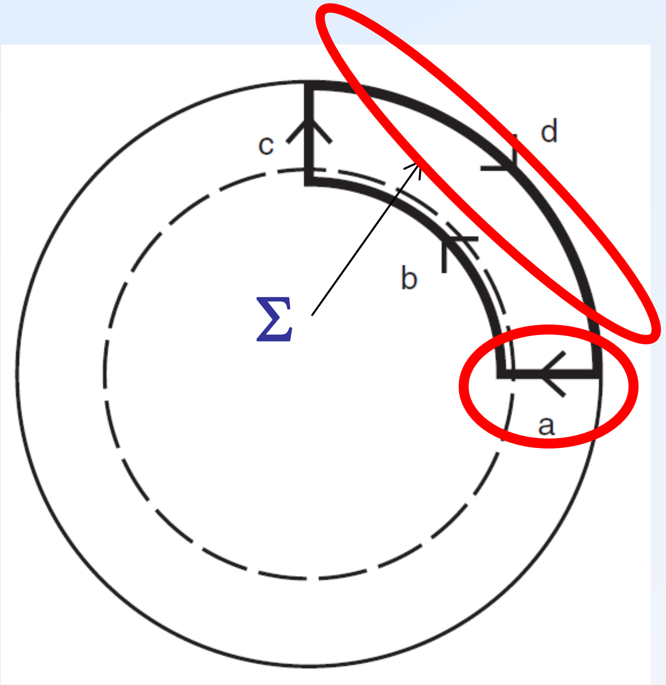


The contribution to Φ_{tor} by radial diff. rotation is a few % of that of latitudinal diff. rotation.

Contribution of radial differential rotation



Compare contributions from parts **a** und **d**



Contribution of radial differential rotation

$$\frac{\mathrm{d}\Phi^{\mathrm{N}}_{\mathrm{tor}}}{\mathrm{d}t} = \int_{\delta\Sigma} (\mathbf{U} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l} = -\int_{r_0}^{R_\odot} \underbrace{U_\phi B_\theta \,\mathrm{d}r}_{\mathbf{a}} + \int_{\pi/2}^{0} \underbrace{U_\phi B_r R_\odot \,\mathrm{d}\theta}_{\mathbf{d}} ,$$

Part a: Assume $5{\times}10^{22}$ Mx poloidal flux threading the NSSL

$$\left|\frac{\mathrm{d}\Phi^{\mathrm{N}}_{\mathrm{tor,NSSL}}}{\mathrm{d}t}\right| \approx 0.5(\Delta U_\phi) B_\theta \Delta r \approx 0.5(\Delta U_\phi)\Phi_{\mathrm{P}}/2\pi R_\odot \approx 1.3 \cdot 10^{22}\,\mathrm{Mx\,yr^{-1}}$$

$$\Delta U_\phi \approx 7.5 \cdot 10^3\,\mathrm{cm\ s^{-1}}$$

Part d: Assume $5{\times}10^{22}$ Mx poloidal flux through 30 deg polar cap

$$\left|\frac{\mathrm{d}\Phi^{\mathrm{N}}_{\mathrm{tor,surf}}}{\mathrm{d}t}\right| \approx (\Delta V) B_r R_\odot \Delta(\cos\theta) \approx (\Delta V)\Phi_{\mathrm{P}}/2\pi R_\odot \approx 1.8 \cdot 10^{23}\,\mathrm{Mx\,yr^{-1}}$$

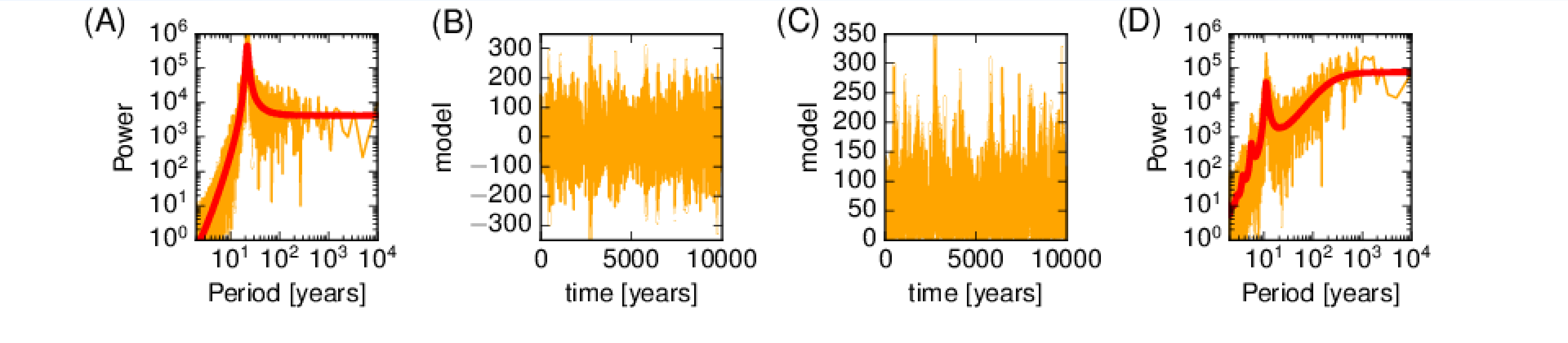
$$\Delta V = |\Omega - \Omega_{\mathrm{eq}}| R_\odot \approx 5 \cdot 10^4\,\mathrm{cm\ s^{-1}}$$

$$\boxed{\left|\frac{\mathrm{d}\Phi^{\mathrm{N}}_{\mathrm{tor,NSSL}}}{\mathrm{d}t}\right| \Big/ \left|\frac{\mathrm{d}\Phi^{\mathrm{N}}_{\mathrm{tor,surf}}}{\mathrm{d}t}\right| \approx 0.07 ,}$$

Surface flux transport (SFT) simulations…

$$\frac{\partial B}{\partial t} = -\Omega(\lambda)\frac{\partial B}{\partial \phi} - \frac{1}{R_\odot \cos\lambda}\frac{\partial}{\partial \lambda}[v(\lambda)B\cos\lambda]$$
$$+ \eta_H \left[\frac{1}{R_\odot^2 \cos\lambda}\frac{\partial}{\partial\lambda}\left(\cos\lambda\frac{\partial B}{\partial\lambda}\right) + \frac{1}{R_\odot^2\cos^2\lambda}\frac{\partial^2 B}{\partial\phi^2}\right]$$
$$+ D(\eta_r) + S(\lambda, \phi, t), \qquad (1)$$

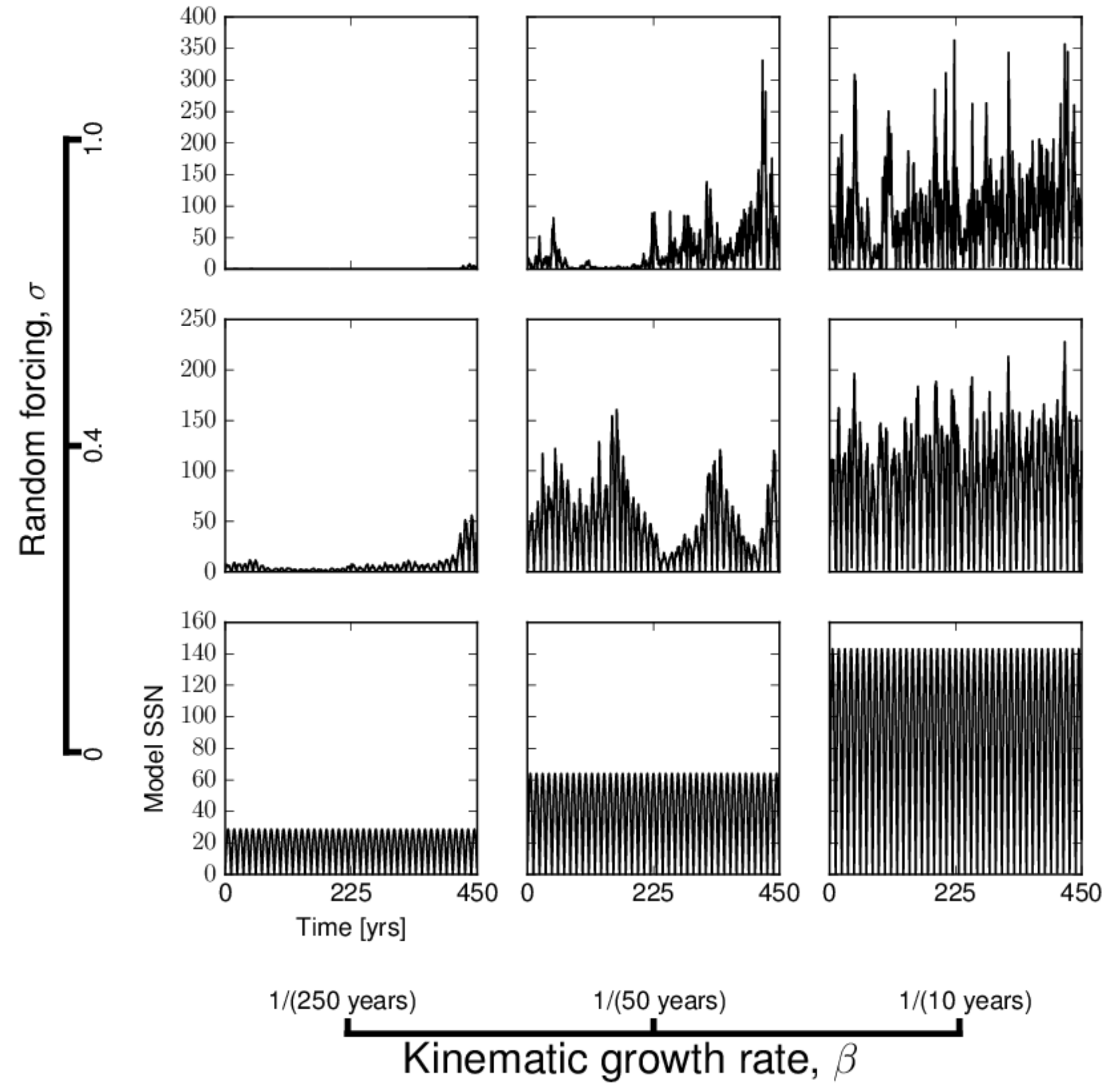
… were rather successful in reproducing the observed (or reconstructed) evolution of polar fields in cycles 15-22, but…



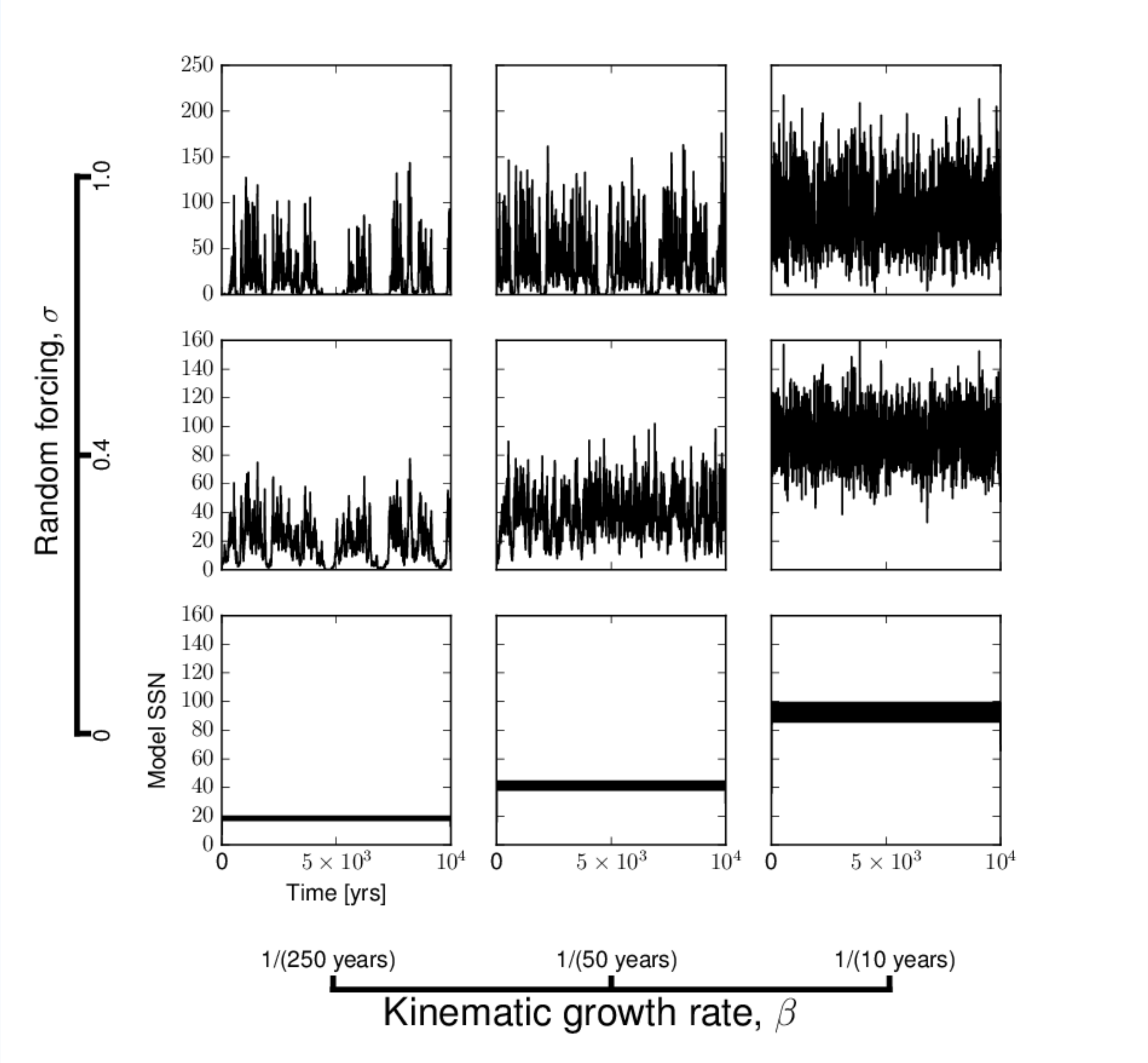
signed quantity

unsigned quantity

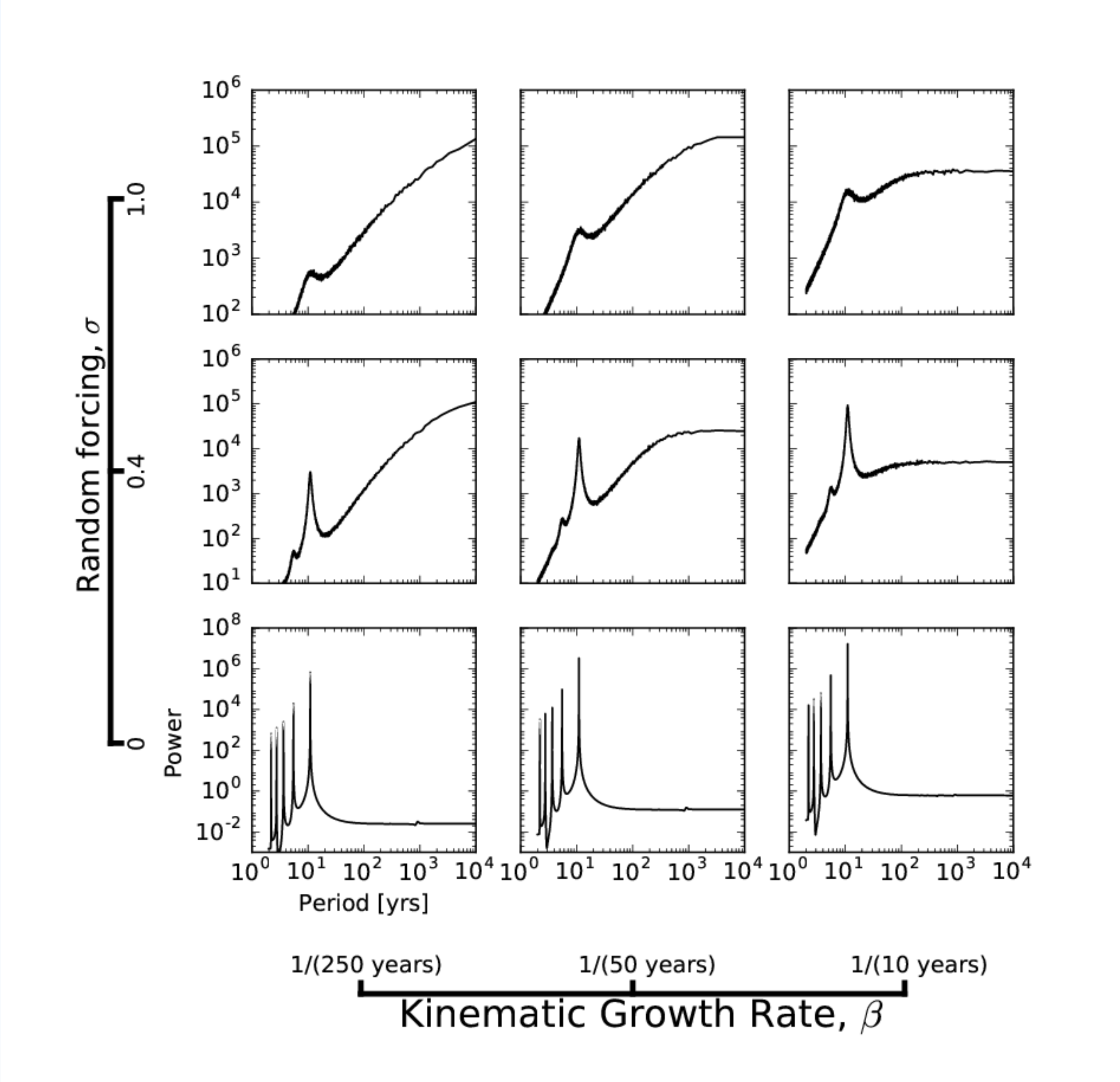
Examples (450 yrs) : normal form model



Examples (10 kyrs) : normal form model



Power spectra: normal form model



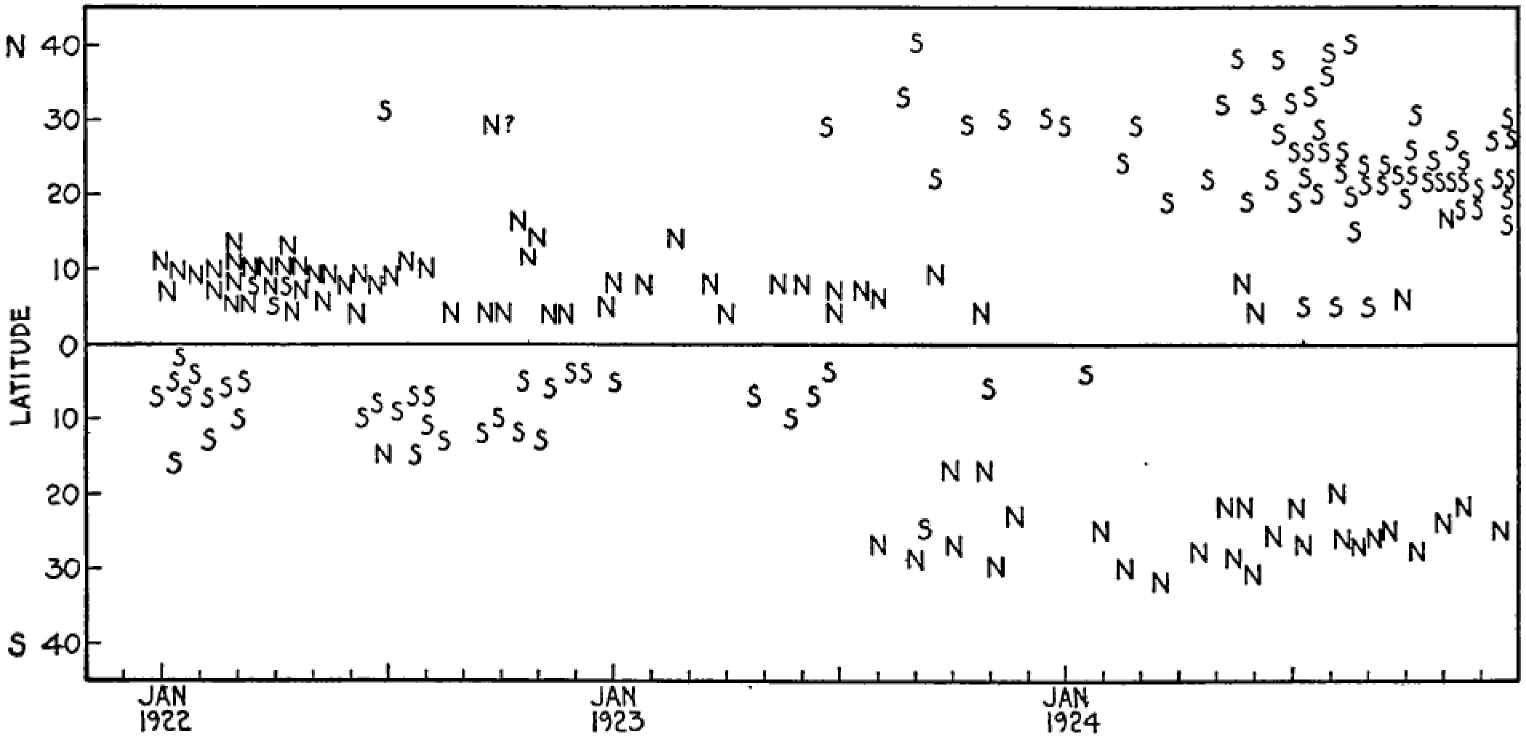


FIG. 16.—Heliocentric latitudes and magnetic polarities of sun-spot groups observed at Mount Wilson from January 1, 1922, to January 1, 1925. *N* (north-seeking) or *S* (south-seeking) represents the polarity of the preceding spot of each group.

Hale & Nicholson (1925)

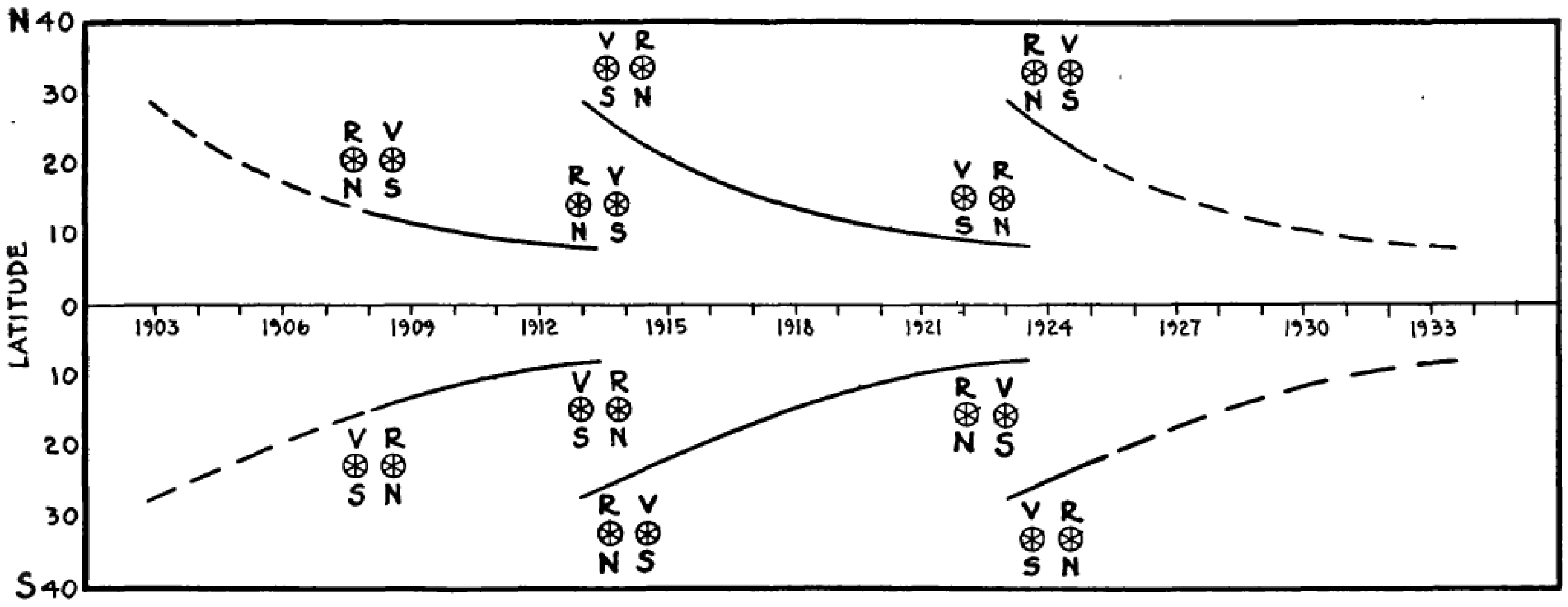


FIG. 18.—The law of sun-spot polarity. The curves represent the approximate variation in mean latitude and the corresponding magnetic polarities of spot groups observed at Mount Wilson from June 1908 to January 1925. The preceding spot is shown on the right.

Hale & Nicholson (1925)

What is the relevant poloidal flux?

$$\frac{\mathrm{d}\Phi_{\mathrm{tor}}^{\mathrm{N}}}{\mathrm{d}t} = \int_{\delta\Sigma} (\mathbf{U} \times \mathbf{B} - \eta_t \nabla \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l}$$

$$\begin{aligned}\int_{\delta\Sigma} (\mathbf{U} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l} &= \int_0^{\pi/2} U_\phi B_r R_\odot \mathrm{d}\theta \\ &= \int_0^1 (\Omega - \Omega_{\mathrm{eq}}) B_r R_\odot^2 \mathrm{d}(\cos\theta)\end{aligned}$$

An analoguous expression is valid for the southern hemisphere.

Result: the amount of net toroidal flux is determined by the surface distribution of emerged magnetic flux and the latitudinal differential rotation.

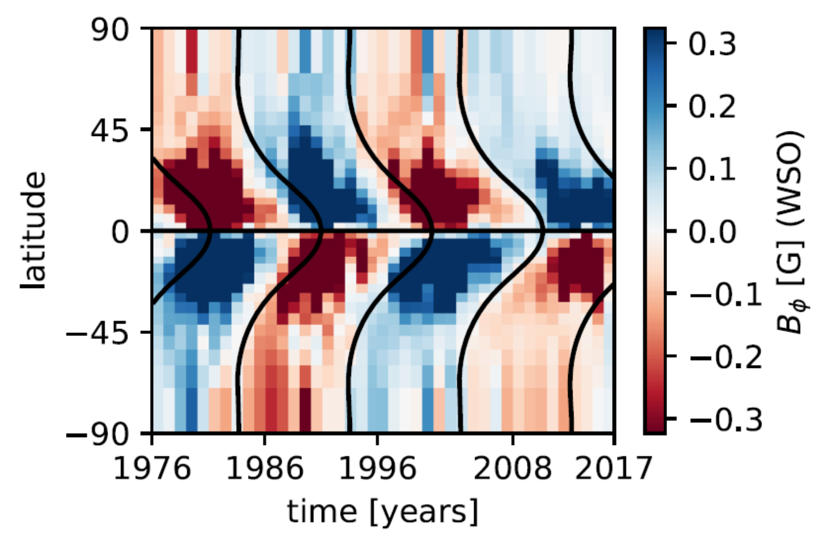
Babcock-Leighton 2.0

Comparison between SFT and 2D flux transport dynamo (Cameron et al., 2012)
→ radial pumping required!

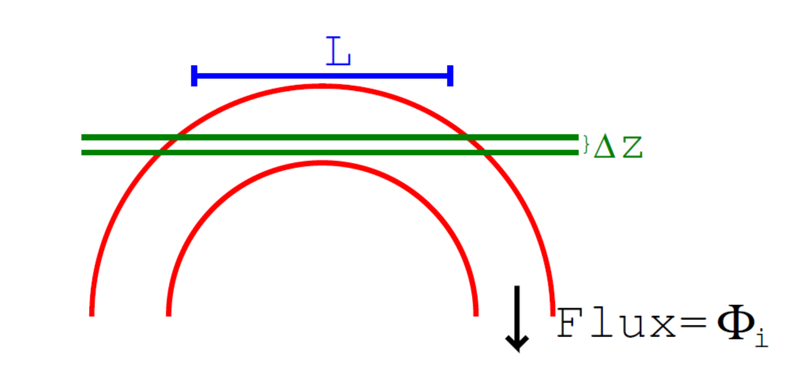
Results:

- solar-like solutions with reasonable parameter values
- consistent with observed toroidal surface field (ref)
- frequencies of N-S asymmetry (ref)

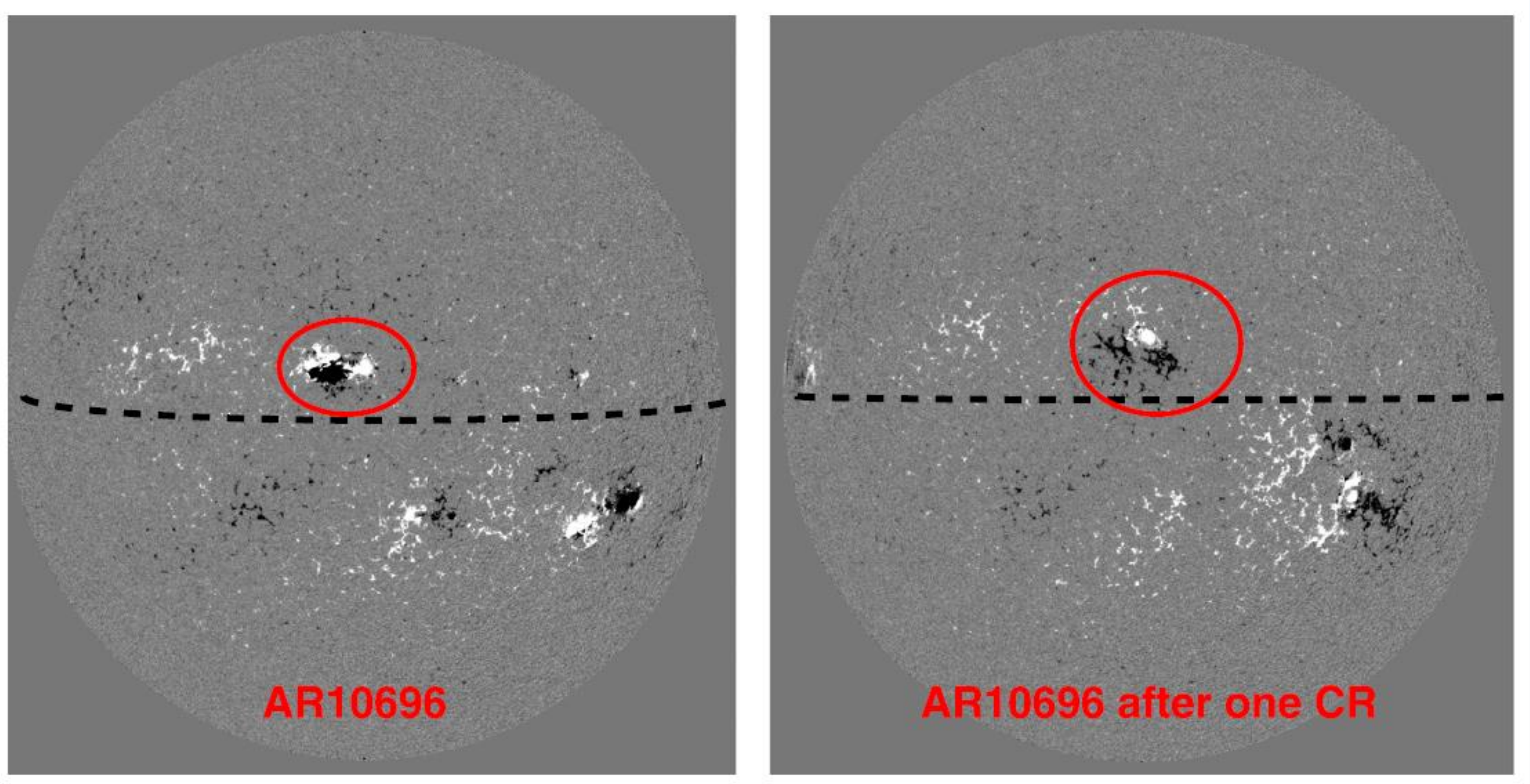
Bϕ at solar surface…



…resulting from flux emergence

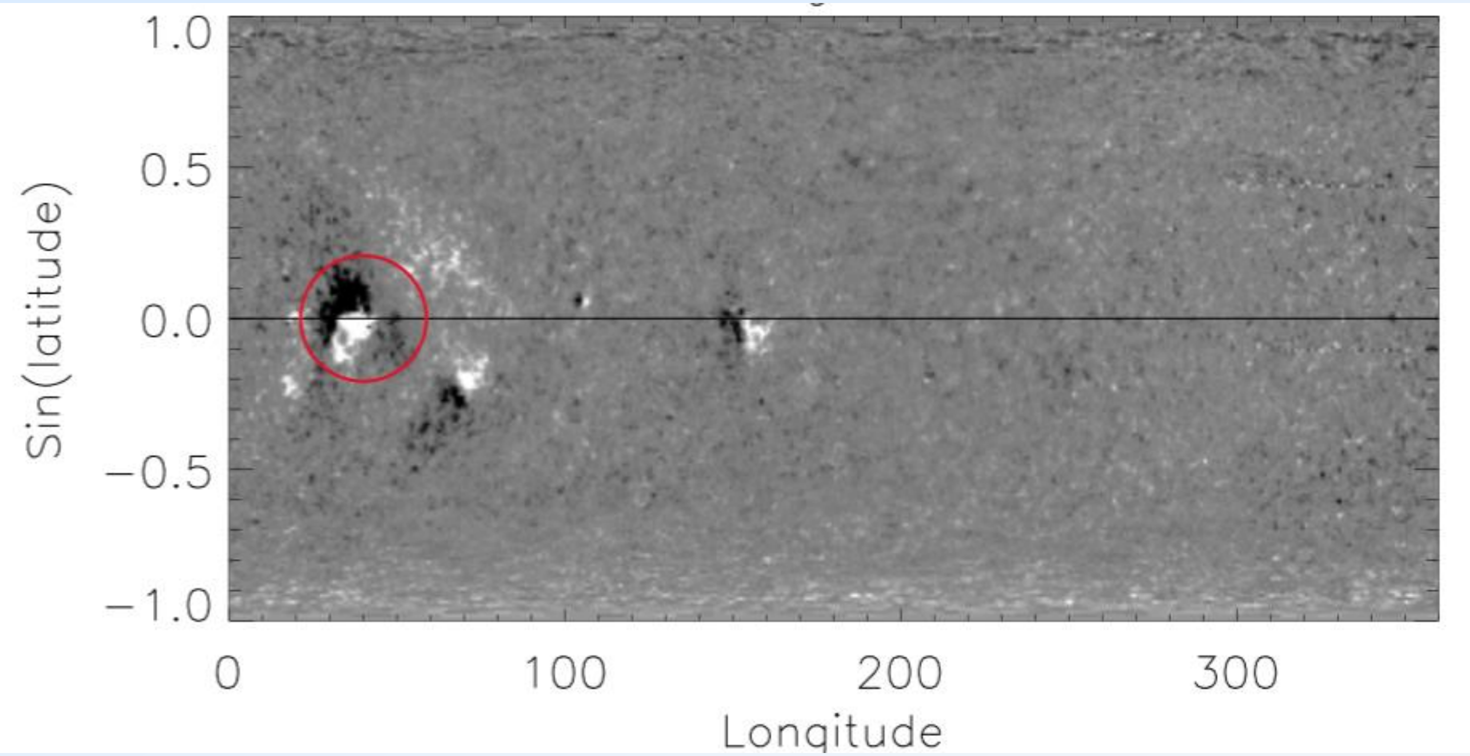


Cameron et al. (2018)



Jiang et al. (2015)

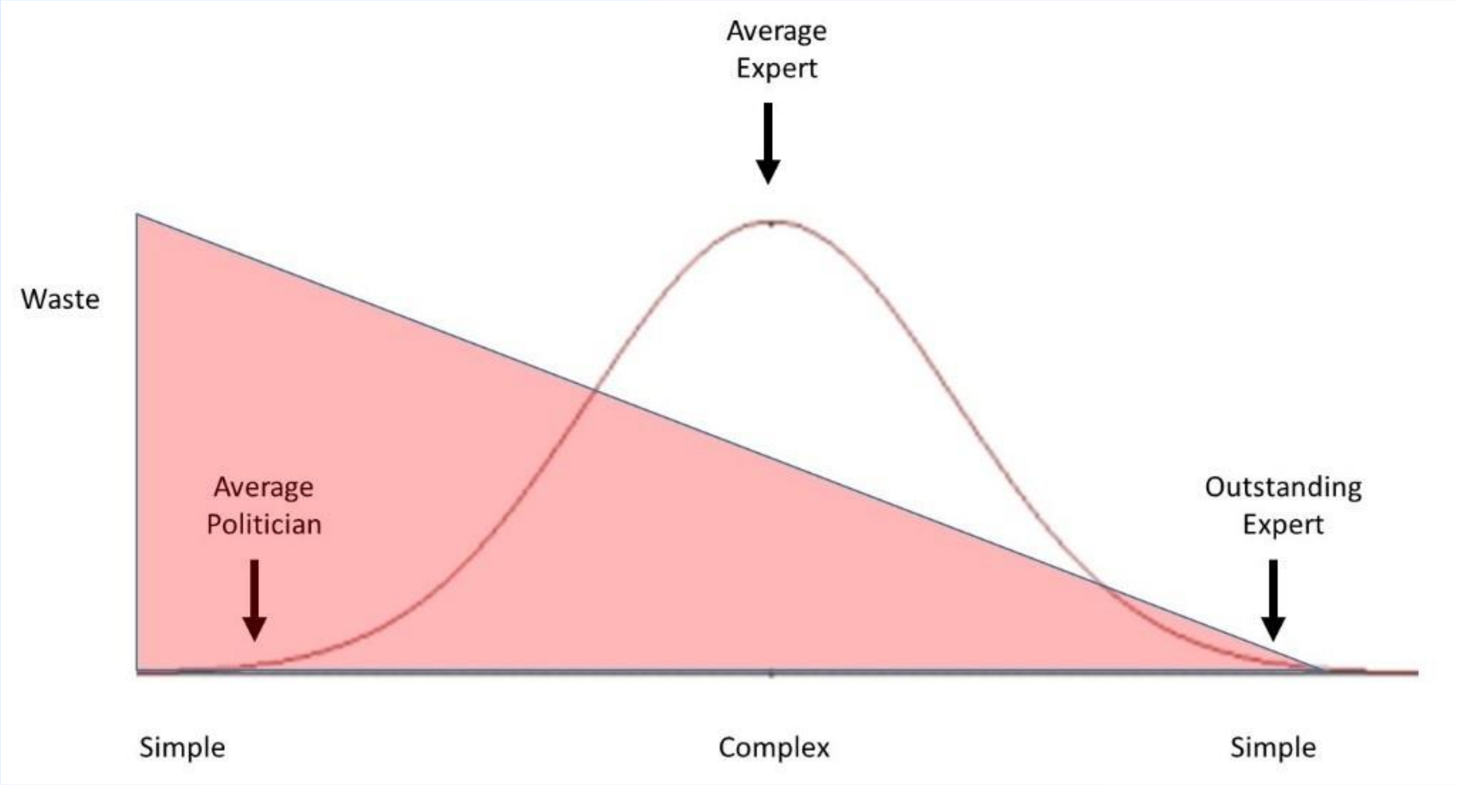
Randomness matters



Cameron et al. (2012)

Kitt Peak synoptic magnetogram for CR 1772 (February 1986)

Single bipolar regions emerging near or across the equator can have a significant impact on the built-up of the polar flux.

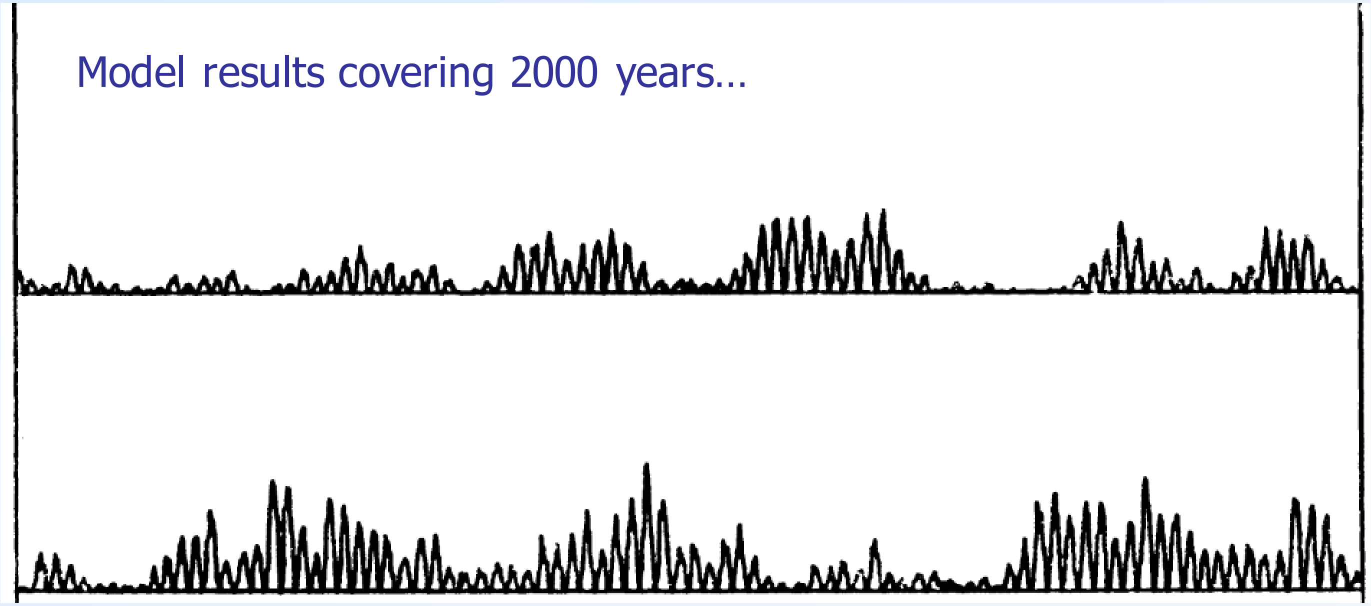

Average
Expert
Waste
Average
Politician
Outstanding
Expert
Simple
Complex
Simple

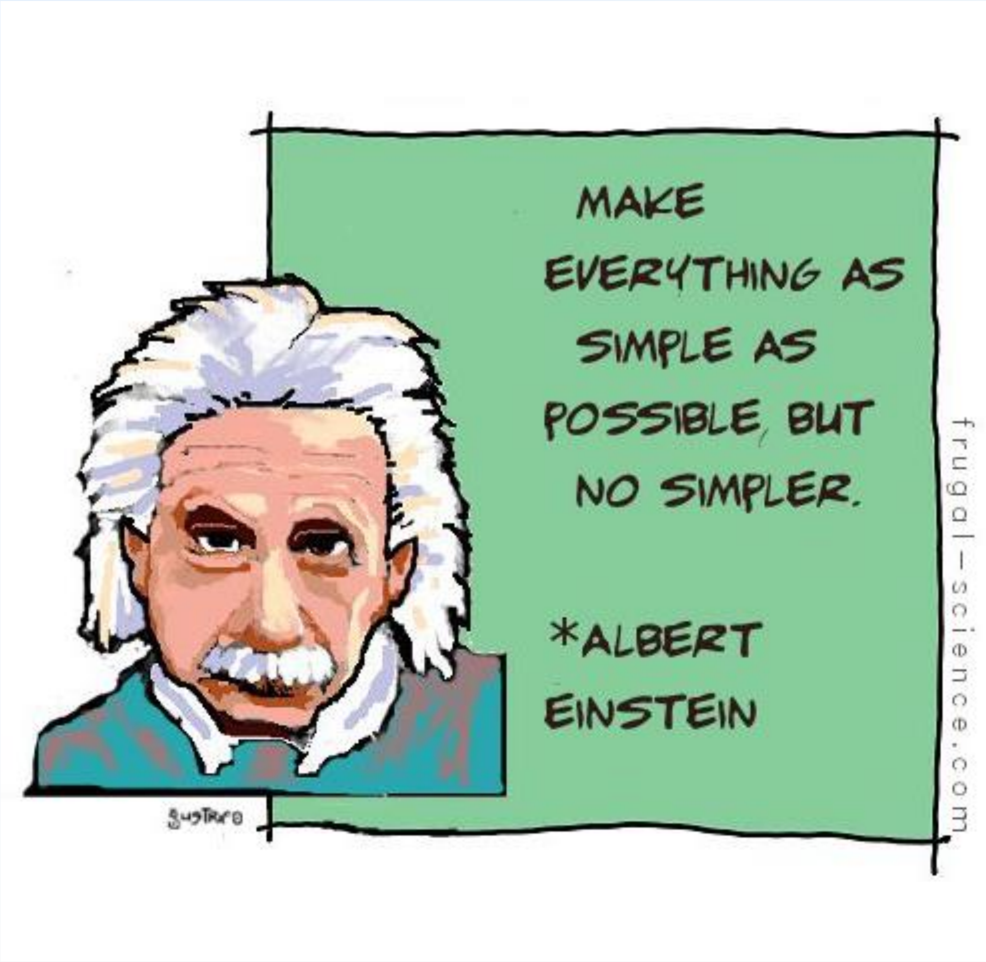
The simplest solar cycle model ever…

Barnes et al. (1980):

Auto-regressive moving-average (ARMA) model (iterative map)
$\rightarrow$ white noise filtered around 1/22 cyc/year with bandwith 0.002 cyc/year

Long-term evolution:





Is there anything we could learn from such a 'model'?

Perhaps yes: **Randomness could be important for the variability of the solar cycle**

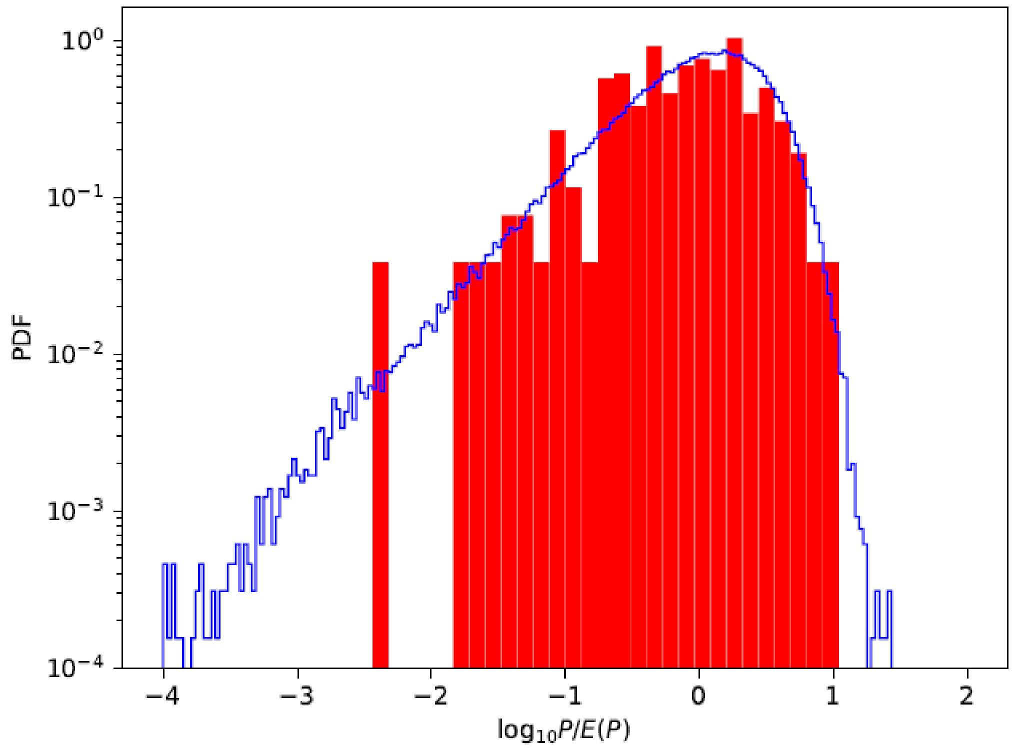
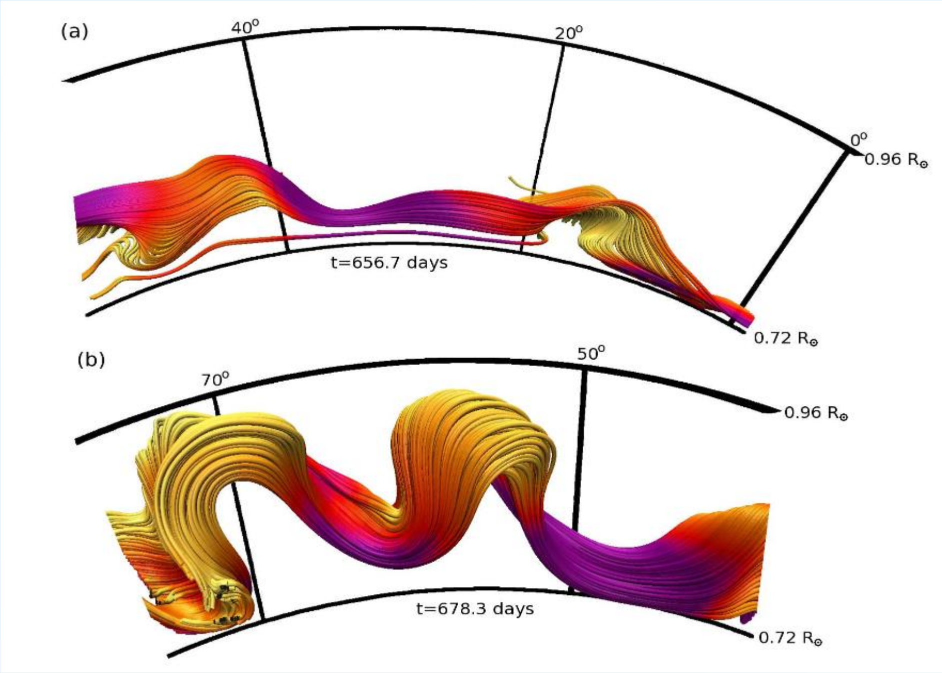
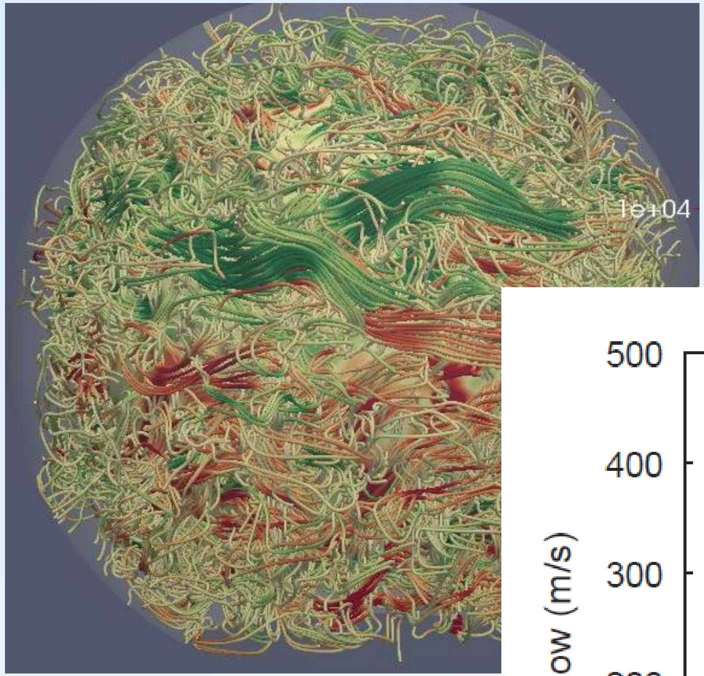


Fig. 3. Probability distribution function of the ratio of the power in a frequency divided by the median power of the NNF model realizations at that frequency. The blue curve is the expectation value based on 1000 realizations of the model, the red bars correspond to the reconstruction from cosmogenic isotopes.

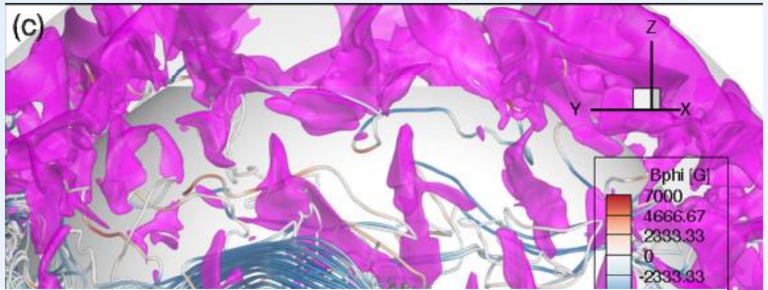
Super-equipartition rising flux loops

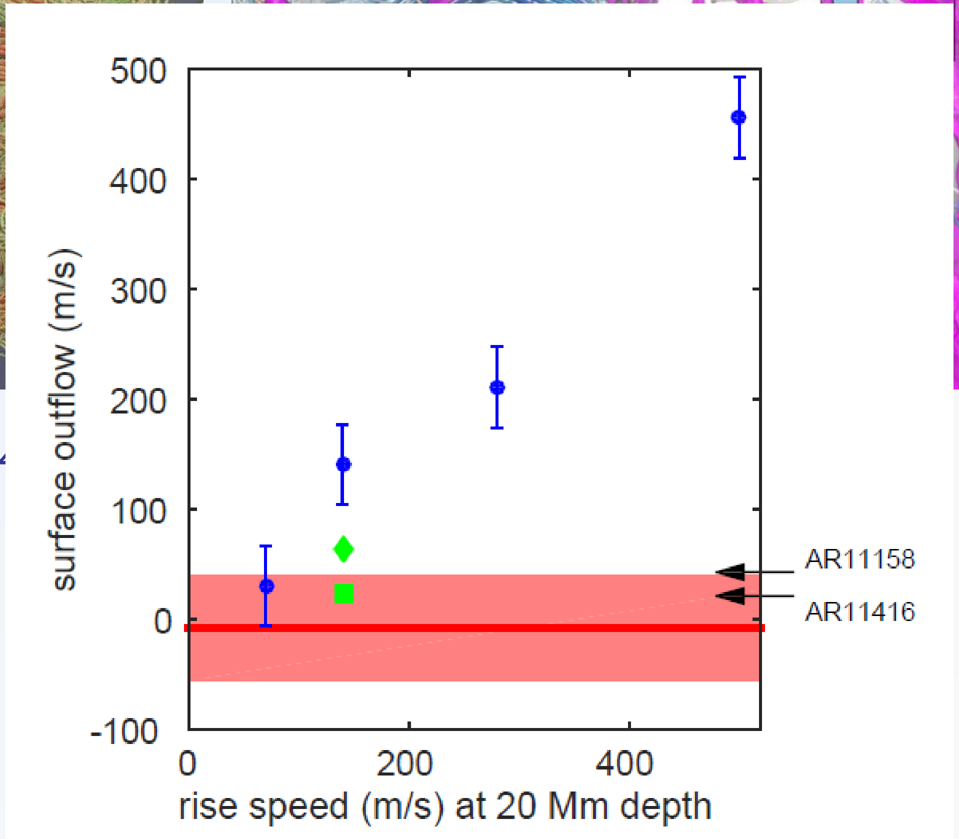


Nelson & Miesch (2014)



Fang & Fan (2014

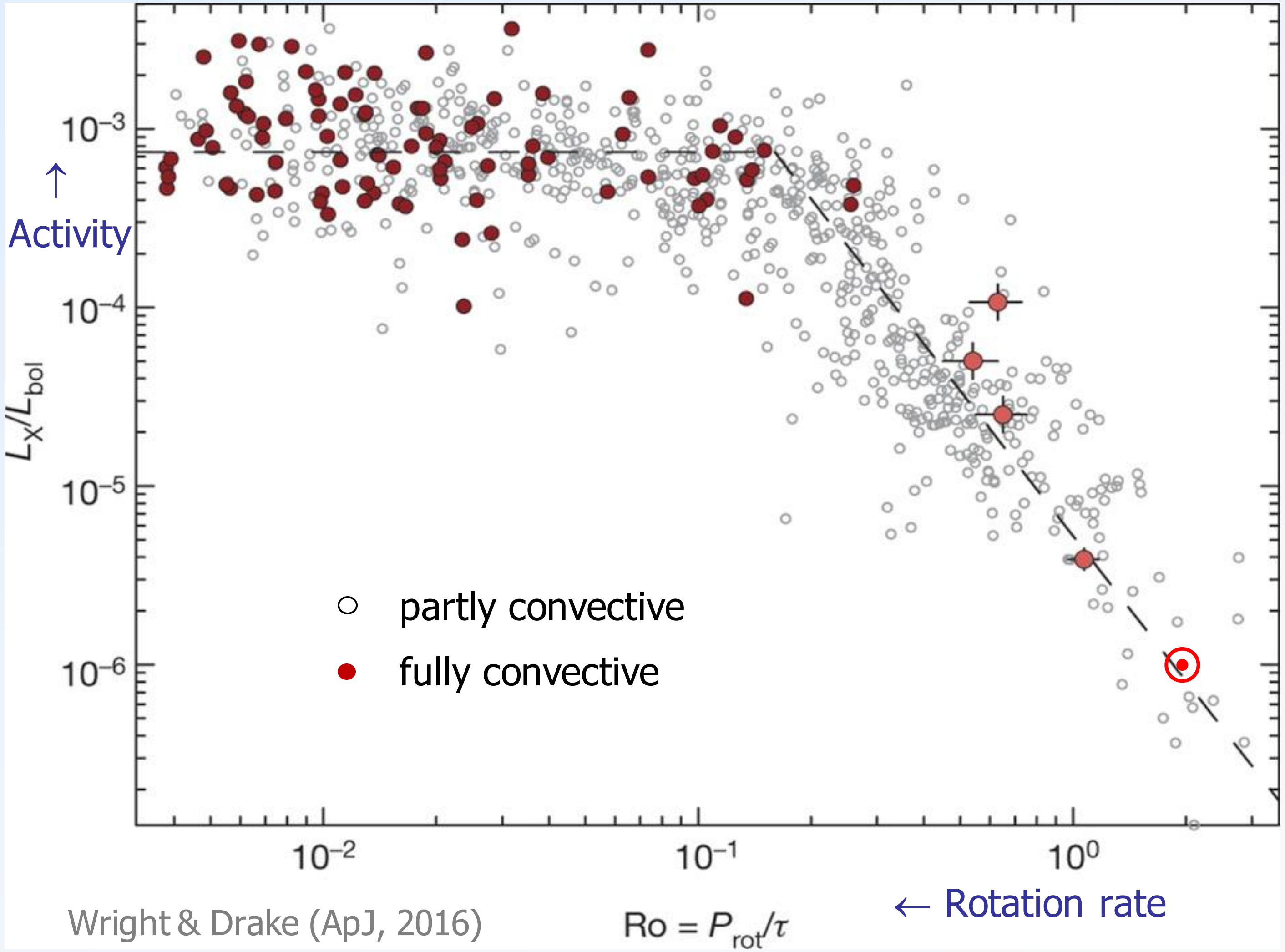




Birch et al. (2016): rise speed of flux loops consistent with convective velocities down to 20 Mm depth

Tachocline: is it relevant for the dynamo?

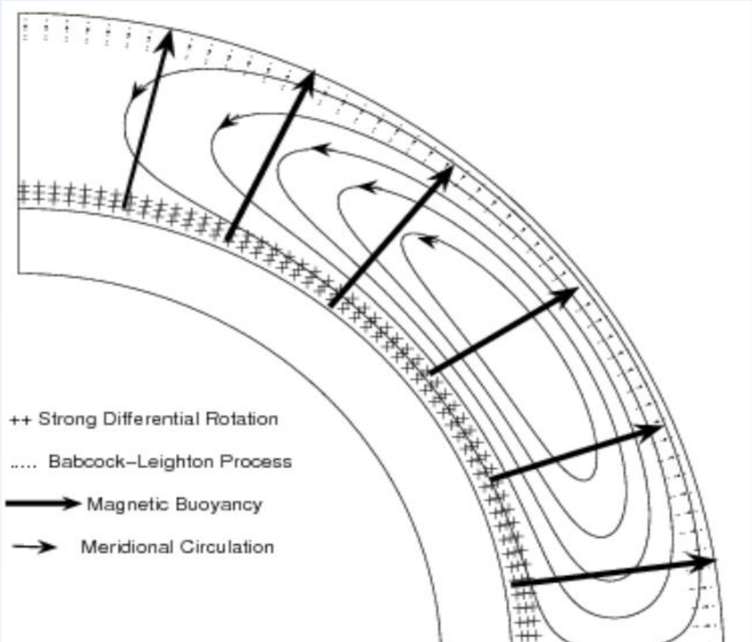
- Absence of a significant cycle variability of tachocline rotation
  ($E_{kin} \approx E_{mag}$ for B ~ 10^5 G; Rempel, 2006)
- Maintainance of a magnetic tachocline?
  (Spruit, 2010)
- Toroidal flux generated by latitudinal differential rotation from flux of the polar field is sufficient to supply the emerged flux
  (Cameron & S., 2015)
- Strong toroidal field bands in the bulk of the convection zone and emergence of loops are exhibited by 3D MHD simulations
  (e.g., Nelson & Miesch, 2014; Fan & Fang, 2014)
- Activity cycles shown by ultracool, fully convective dwarfs (≥ M7)
  (Route, 2016)
- Partly and fully convective stars follow the same activity-rotation law
  (Wright & Drake, 2016)



Wright & Drake (ApJ, 2016)

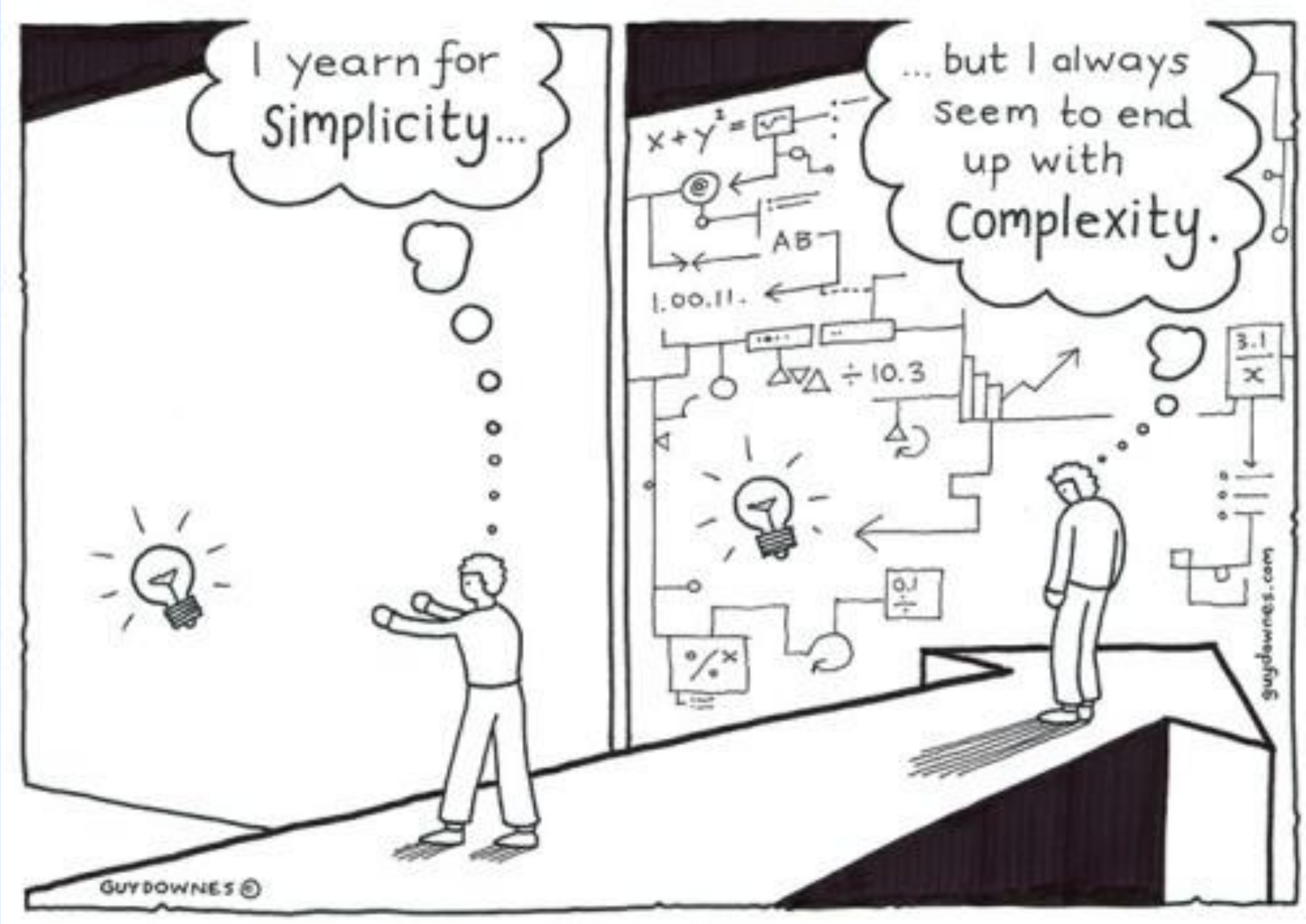
Challenges to the „current paradigm"

- 3D MHD simulations show cyclic dynamo action within the convection zone (without tachocline, overshoot layer, etc.)
- Super-equipartion fields and rising flux loops may form within the convection zone
- Maintainance of tachocline differential rotation against magnetic stresses (Rempel, 2006; Spruit, 2010)?
- Existence of a sufficiently extended & subadiabatic 'storage region' (e.g. Hotta, 2017)?
- Fully convective stars fit well in the activity-rotation relations



(Karak et al., 2014)

A solar physicist's lament



- How long will we have to wait until we have a reliable 3D-MHD simulation of the solar cycle?
- Do the non-simulators need to sit idle and wait until then?
- Or can we still learn something useful in the meantime through observations, theory & simple models?