Long-term Variation of the Coupling between Solar Proxies: Coupled Oscillators Approach

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Outline

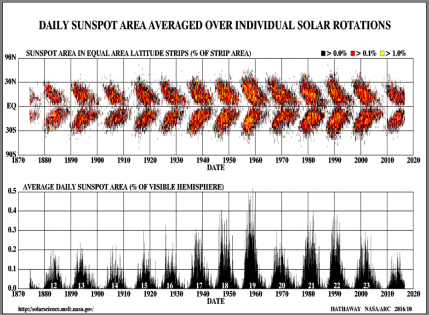
Introduction: cyclic scenarios and the meridional flow

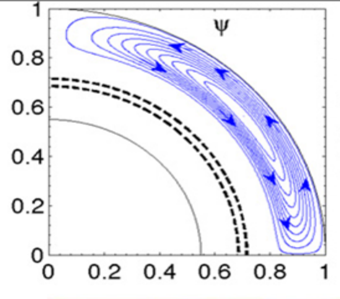
Reconstructions in Kuramoto model

Advances: time-dependent coupling and Kuramoto's applicability

Advances: model misinterpretation — van der Poll-Duffing case

Intro: Oscillating Trend in Solar Activity





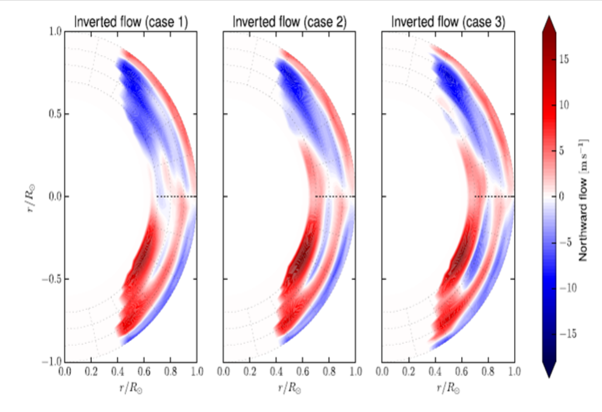
Hazra et al, 2014

Idea

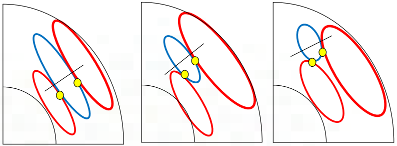
Independently of physical background one should try to reproduce phenomena of the solar dynamo with models of interacting oscillators.

Intro: Meridional Flow and Cells Structure

Boning et al, 2017: GONG reconstructions 2004-2012



- *Three* cells of the meridional flow per hemisphere are assumed;
- Their placement is up to debate;



- Each cell is associated with an oscillator; one should consider their interactions (couplings).

Oscillators' Modelling: Kuramoto

Kuramoto Model of the Coupled Oscillators

Oscillators: $X_i(t) = A_i(t) \sin\left(\Omega_i t + \varphi_i\right)$; $\theta_i = \Omega_i t + \varphi_i$ are phases

$$\dot{\theta}_i = \underbrace{\omega_i}_{\text{natural frequencies}} + \sum_j \underbrace{\kappa_{ij}}_{\text{coupling}} \sin(\theta_j - \theta_i), \quad \Omega_i \text{ are frequencies}$$

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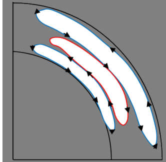

- There-layer structure of the meridional flow

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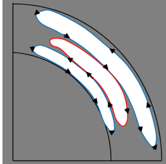

- There-layer structure of the meridional flow
- Relationship between the inner and outer layers is organized through the middle layer

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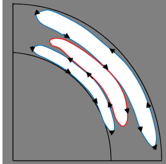

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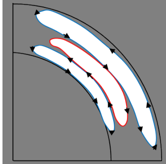


- There-layer structure of the meridional flow
- Relationship between the inner and outer layers is organized through the middle layer
- The coupling is symmetrical, $\kappa_{ij} = \kappa_{ji}$
- Oscillators are synchronized: $\dot{\theta}_i - \dot{\theta}_j = 0$

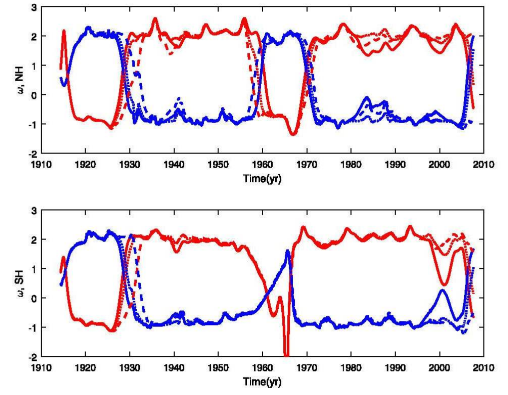
Oscillators' Modelling: Reconstruction in Kuramoto

- Two oscillators $X_1(t)$ and $X_2(t)$ given by the toroidal and poloidal magnetic fields and represented, f. i., by solar proxies ISSN and aa
- Single equation: $\boxed{\dot{\theta}(t) = 2\Delta\omega - \kappa(t) \sin\theta(t)}$, where $\theta = \theta_1 - \theta_2$.
- Correlation $C_1(t) = \big(\textbf{Corr}(X_1(t), Y_1(t))\big)$ is computed over the period, say, $T = 10.75$ years with the sliding windows
- Estimate φ of the phase θ: $\boxed{\varphi(t) = \arccos C_1(t)}$, Blanter et al (2014)
- Synchronization $\boxed{\dot{\theta} = 0} \Rightarrow \boxed{2\Delta\omega - \kappa(t) \sin\varphi(t) \approx 0}$
- The estimate of the coupling

$$\kappa(t) = \frac{2\Delta\omega}{\sin\varphi(t)}$$

A Reconstruction of Natural Frequencies

Kuramoto Model with Three Oscillators



Blanter et al. 2018

frequencies ω of
top (in red)
and
bottom (in blue)
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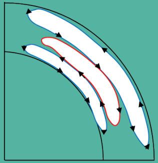
in the northern and
southern hemispheres



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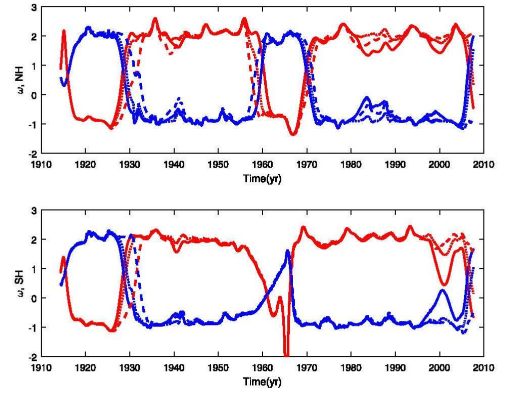

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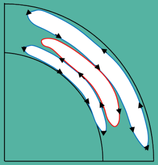

- Faster velocity of the surface cell and lower velocity of the deep cell

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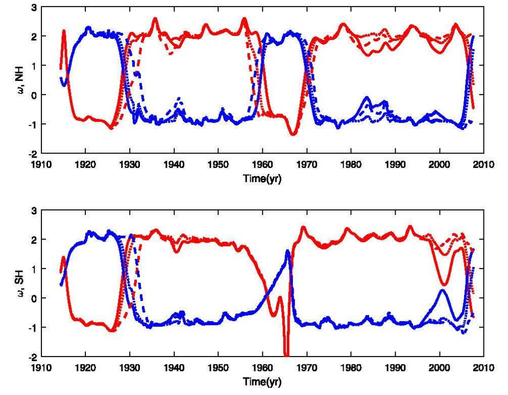

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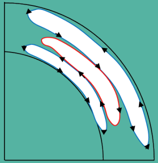


- Faster velocity of the surface cell and lower velocity of the deep cell
- The opposite regime: the 1920s, the late 1960s, and the 2000s–

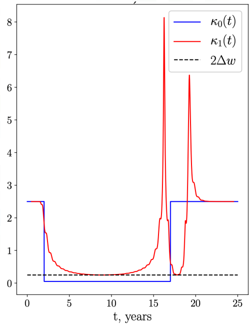
Kuramoto Reconstruction: time-dependent coupling

Time-dependency

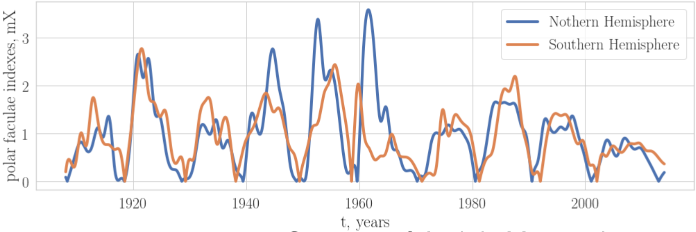
- We move from constant couplings κ to time-dependent $\kappa(t)$;
- Earlier there were *no* synchronisation if $\Delta\omega > \kappa$; now temporarily breaks are allowed.

Derived regularities

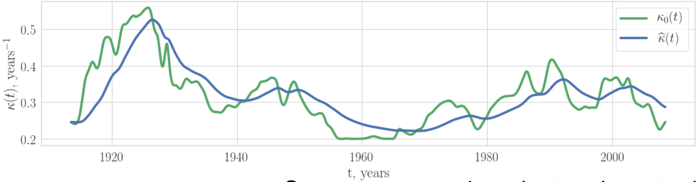
- rapid changes in initial coupling results in long relaxation time of reconstructed coupling
- long breaks of the synchronisation inequality results into complete reconstruction failure (fig.)
- Adding AR(1) noise could lead to this failure (the probability depends on autocorrelation and deterministic coupling)



Kuramoto Reconstruction: time-dependent coupling



Courtesy of Andrés Muñoz–Jaramillo

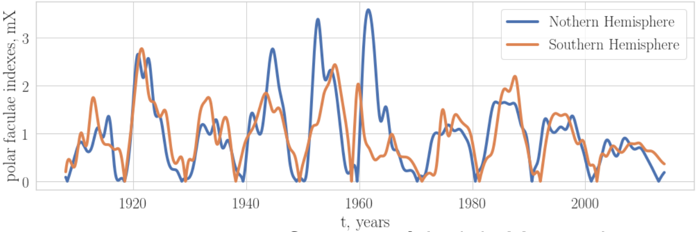


Savostianov et al., submitted, revised
and went to the farm where all old papers go

Example for solar data

- Reconstruction procedure applied to *polar faculae* data;
- Initial coupling extracted by quasi-stationary assumption; the coupling is reconstructed one;

Kuramoto Reconstruction: time-dependent coupling



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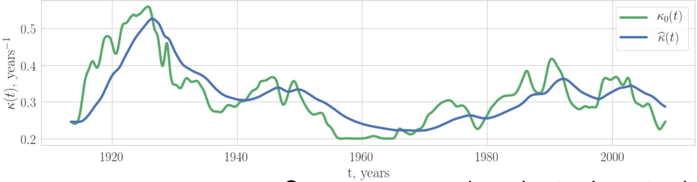

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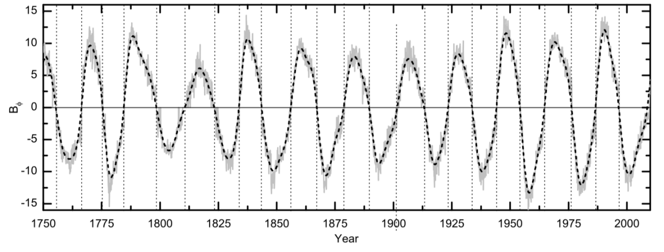
Example for solar data

- Reconstruction (except the 20th cycle) is successful;
- With addition of AR(1) noise, the 20th cycle has a high probability of reconstruction failure.

Oscillators' modelling: van der Poll-Duffing Oscillator

- Lack of solar physical background in case of Kuramoto oscillators;
- More comprehensive (no sine-like) form of solar cycles;
- Can be derived from MHD equations (+ assumption of axysymmetry)

$$\frac{d^2 B_\phi}{dt^2} + \omega^2 B_\phi + \mu(3\xi B_\phi^2 - 1)\frac{dB_\phi}{dt} - \lambda B_\phi^3 = 0$$

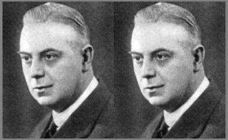


Lopes et al. 2015

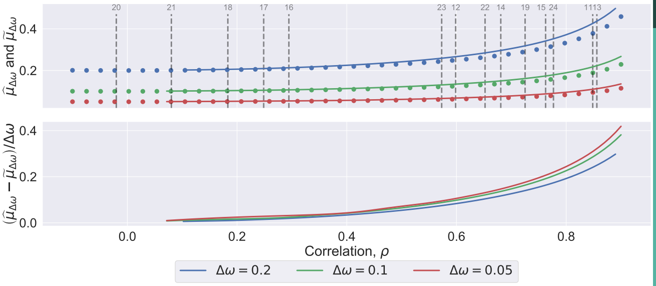
Balthasar van der Pol

From Single VPD to Coupled VPD

Coupled van der Poll-Duffing



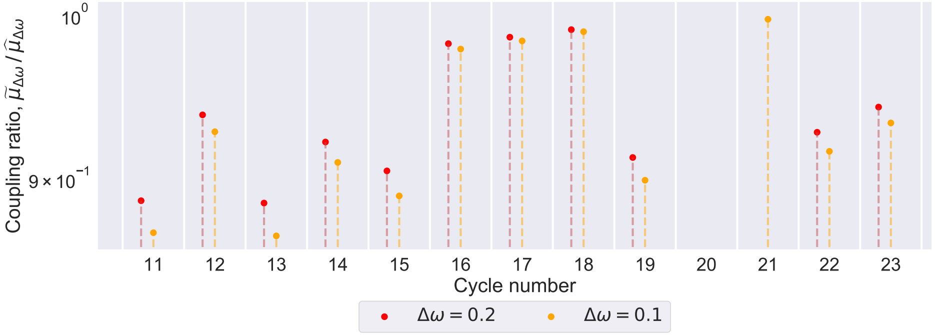
$$\begin{cases} \ddot{x} - (\lambda_1 - x^2)\dot{x} + (1 - \Delta\omega)x + \beta x^3 - \mu(\dot{x} - \dot{y}) = 0 \\ \ddot{y} - (\lambda_2 - y^2)\dot{y} + (1 + \Delta\omega)y + \beta y^3 - \mu(\dot{y} - \dot{x}) = 0 \end{cases}$$



Idea

- Numerically establish correlation/coupling relation for VPD model
- Extract VPD coupling from correlation/coupling relation per solar cycle
- Reconstruct with Kuramoto

Models' Misinterpretation Bias in Reconstruction



Results

- Reconstruction is not entirely catastrophic (error does not exceed $\sim 10\%$)
- The 20th cycle remains to be anomaly
- Proper way to choose the natural frequency level ($\Delta\omega$) is crucial

Thank you for attention

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Contact someone older: abshapoval@gmail.com