SOLAR CYCLE PREDICTION ON YOUR FINGERS (AND TOES)

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european solar telescope

In the forest of space climate prediction:

For a map of the forest, check the 2019 revision of my Living Review:
`https://arxiv.org/abs/1907.02107`

Table 2 A selection of early forecasts for Cycle 25

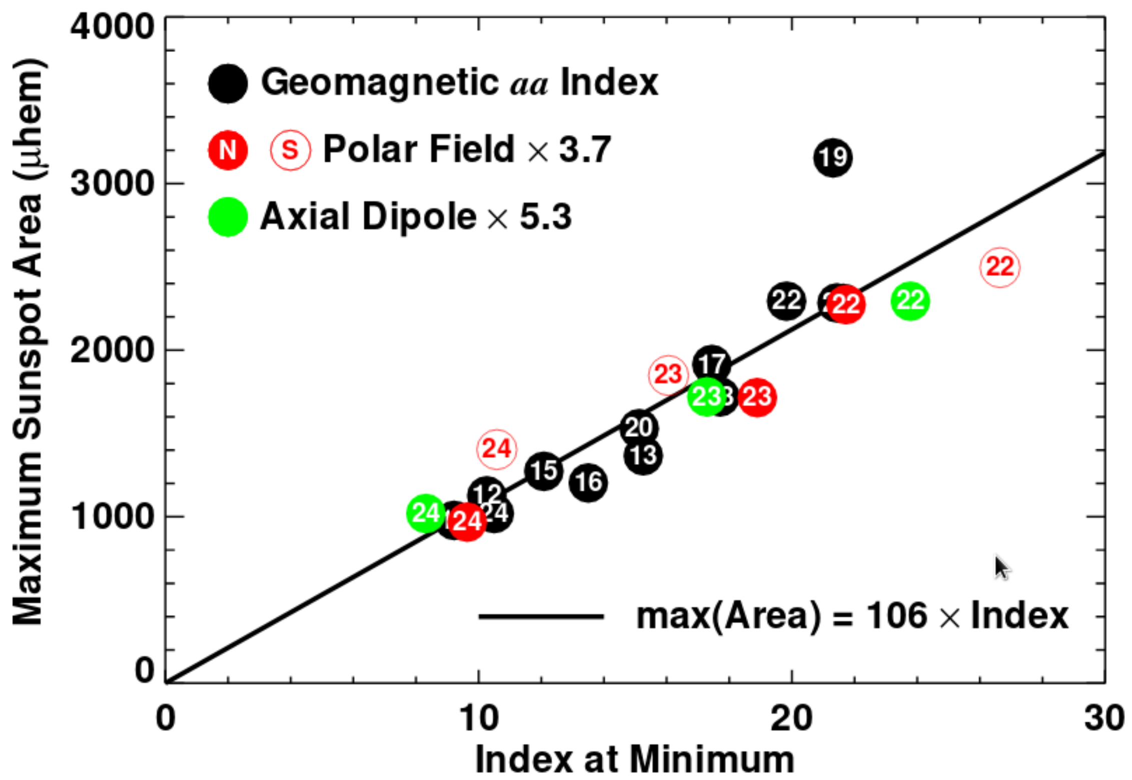
Category	Minimum	Maximum	Peak amplitude	Reference
Internal precursors	2019.9	2023.8	175 [154–202]	Li et al (2015)
External precursor				
rush-to-the-poles	2019.4	2024.8	130	Petrovay et al (2018)
polar precursor			136 ± 48	Pesnell and Schatten (2018)
helicity			117	Hawkes and Berger (2018)
SoDA		2025.2 ± 1.5	120 ± 39	based on Pesnell and Schatten (2018)
Model-based: SFT				
SFT			124 ± 31	Jiang et al (2018)
AFT	2020.9		95 % of Cycle 24, i.e. 110	Upton and Hathaway (2018)
Model-based: dynamo				
2×2D	2020.5 ± 0.12	2027.2 ± 1.0	89^{+29}_{-14}	Charbonneau et al. 2019, private comm.
Truncated	2019–20	2024 ± 1	90 ± 15	Kitiashvili (2016)
Spectral				
wavelet decomposition tree		2023.4	132	Rigozo et al (2011)
Attractor analysis				
simplex projection analysis		2024.0 ± 0.6	103 ± 25	Singh and Bhargawa (2017)
simplex proj./time-delay		2023.2 ± 1.1	154 ± 12	Sarp et al (2018)
Neural networks				
neuro-fuzzy		2022	90.7 ± 8	Attia et al (2013)
spatiotemporal		2022–23	57 ± 17	Covas et al (2019)
Cycle 24 [comparison]	2008.9	2014.3	116	

In this talk, we'll simply take a walk...

OUR BEST BET: THE POLAR PRECURSOR

Observations || dynamo concept:

Polar field at minimum ⇒ amplitude of next maximum



Hathaway & Upton (2016)

Key issue: What determines the value of the precursor?

Polar field builds up from poleward transport of *unbalanced* trailing polarity AR fields, described by surface flux transport (SFT) models.

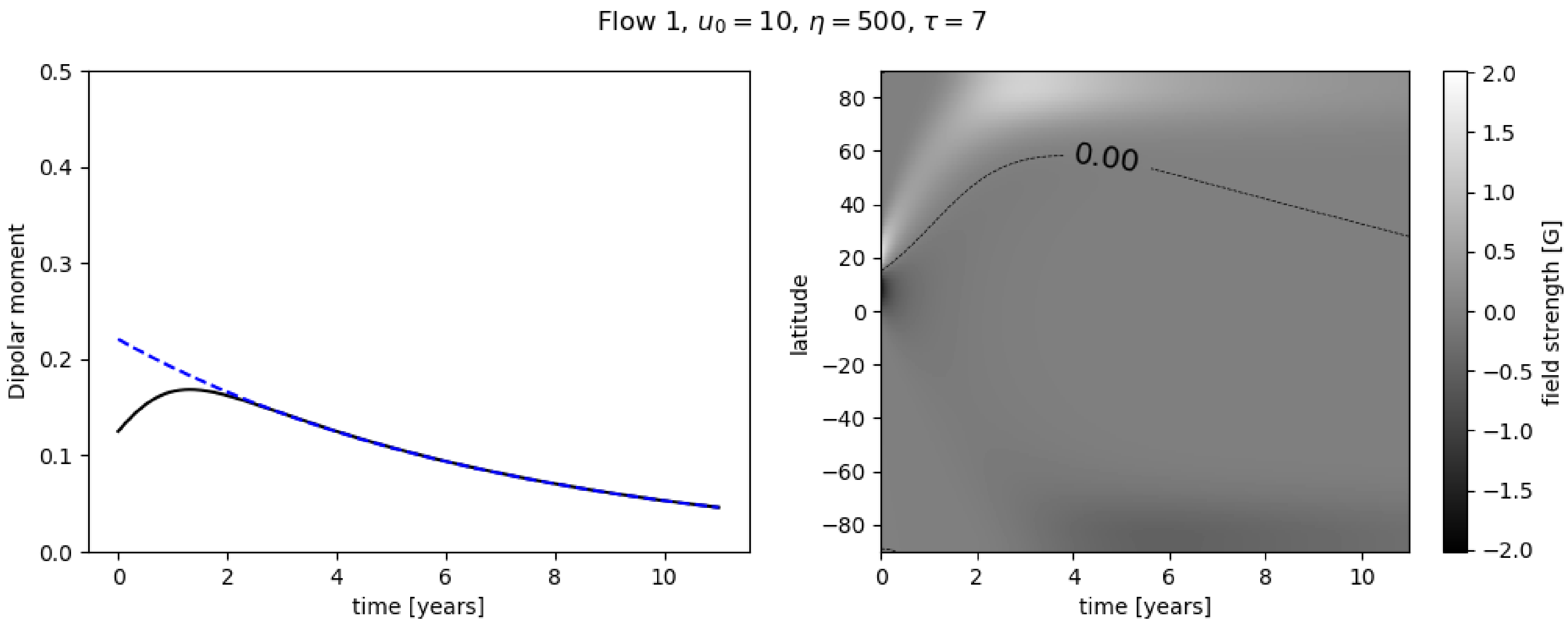
SFT equation:

$$\frac{\partial B}{\partial t} = -\Omega(\lambda)\frac{\partial B}{\partial \phi} - \frac{1}{R\cos\lambda}\frac{\partial}{\partial\lambda}[B\,u(\lambda)\cos\lambda]$$

$$+\frac{\eta}{R^2}\left[\frac{1}{\cos\lambda}\frac{\partial}{\partial\lambda}\left(\cos\lambda\frac{\partial B}{\partial\lambda}\right)+\frac{1}{\cos^2\lambda}\frac{\partial^2 B}{\partial\phi^2}\right]-\frac{B}{\tau}+S(\lambda,t)$$

Btw. recent evidence for the need of a decay term: Virtanen et al. (2017), Whitbread et al. (2019), Petrovay & Talafha (2019)

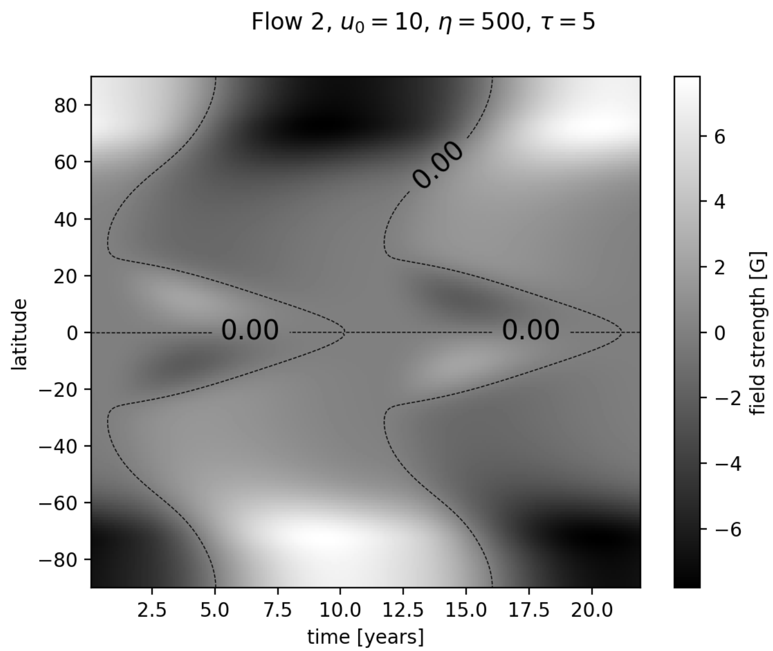
Consider a single AR source:



The SFT equation is linear ⇒ solutions can be superposed ⇒

⇒ polar fields are built up from the contribution of many individual AR:

ARs are responsible for the reversal of the polar field and for the buildup of new, opposite polarity polar field late in the cycle.



Polar fields serve as the seed for the toroidal field in the next cycle ⇒ amplitude of next cycle may be determined well before the minimum by considering the dipole contributions of individual AR. (Wang & Sheeley 1991)

Intercycle variations may be due to

– variations in the meridional flow

(Dikpati et al. 2010, Upton & Hathaway 2014, Hung et al. 2017)

– variations in the unbalanced flux contribution by active regions

Dipole moment:
$$D(t) = \frac{3}{2}\int_{-\pi/2}^{\pi/2} \langle B\rangle^{\phi}(\lambda, t)\sin\lambda\cos\lambda\,\mathrm{d}\lambda.$$

A bipolar AR contributes
$$\boxed{\delta D_i = \frac{3}{4\pi R^2}\Phi\, d\,\sin\alpha\,\cos\lambda}$$

$\Rightarrow$ Variations in number, Φ, λ and tilt of AR lead to intercycle variations.

Variations in AR dipole contribution may be due to

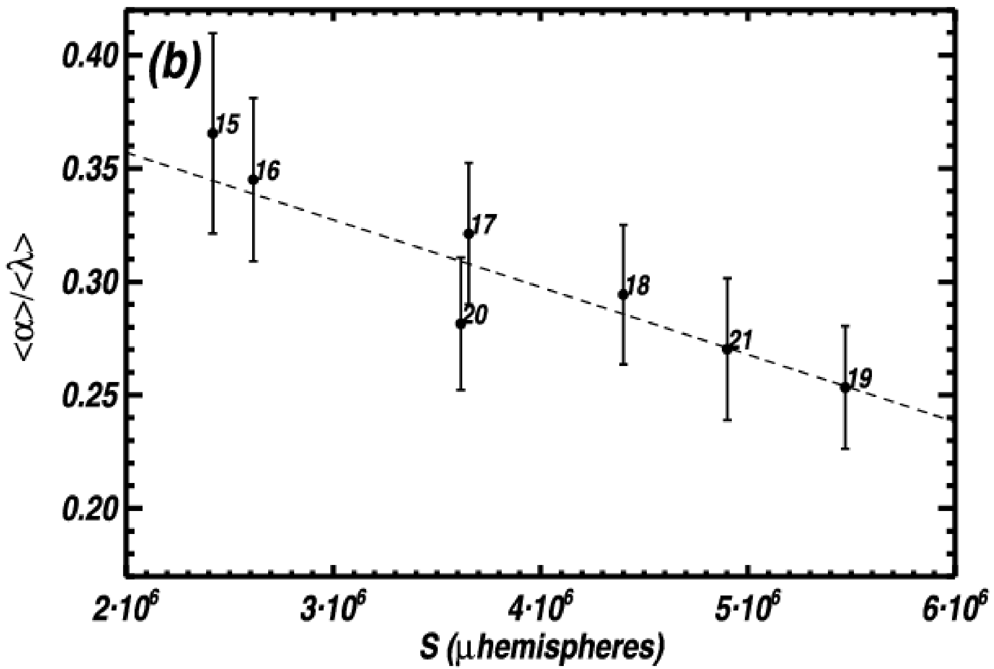
(1) systematic feedback (e.g. tilt quenching)

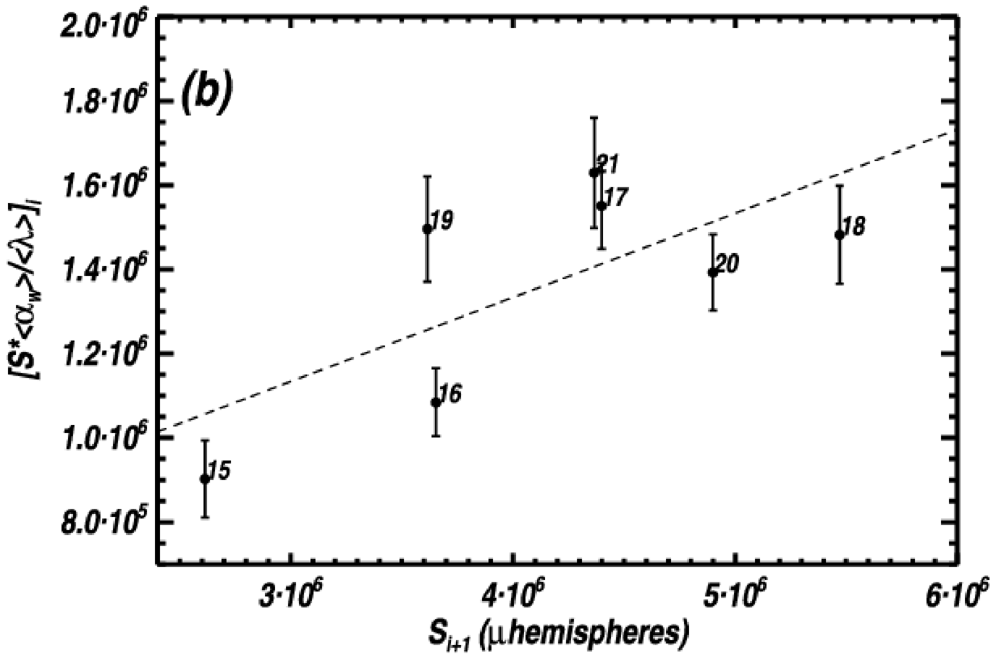
(2) random fluctuations

TILT QUENCHING — TILT PRECURSOR

Dasi-Espuig et al (2010):

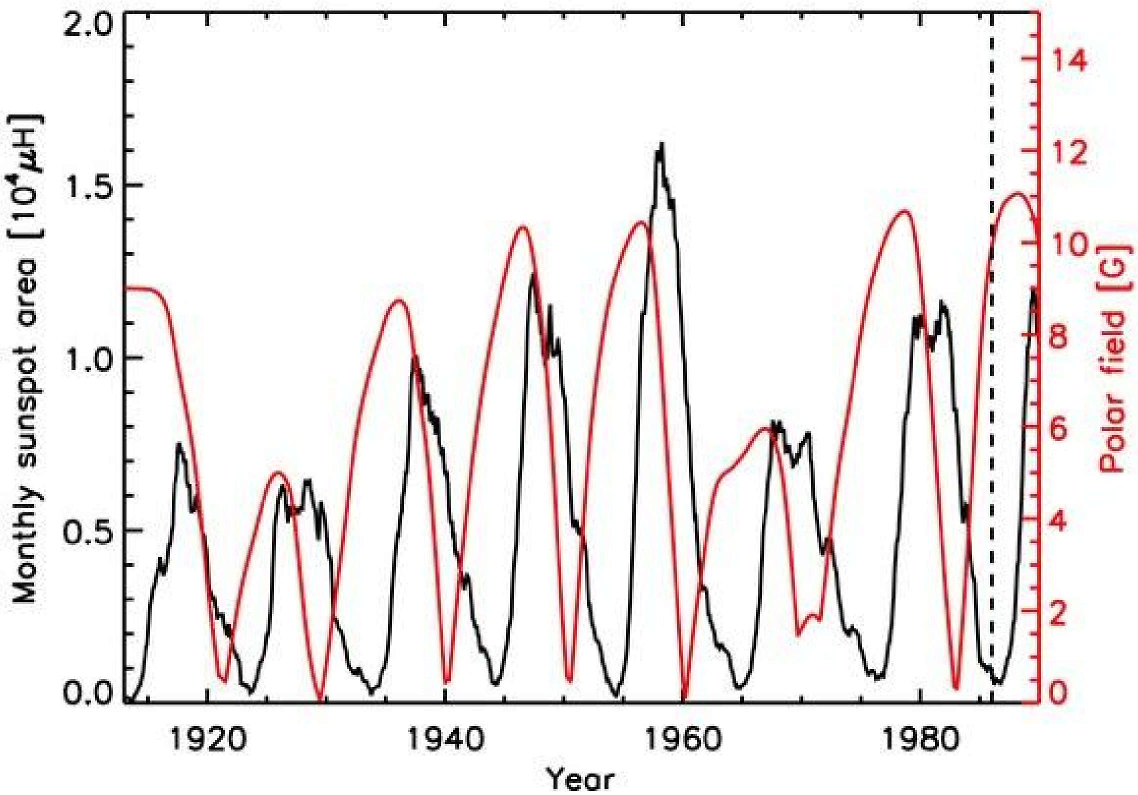
(a) Stronger cycles – lower tilt. (b) Tilt $\times$ amplitude $\Rightarrow$ next cycle ampl.





Gives rise to idea of “tilt quenching” — a nonlinear feedback mechanism governing cycle to cycle variations.

Effect incorporated into SFT model: Cameron et al. (2010)



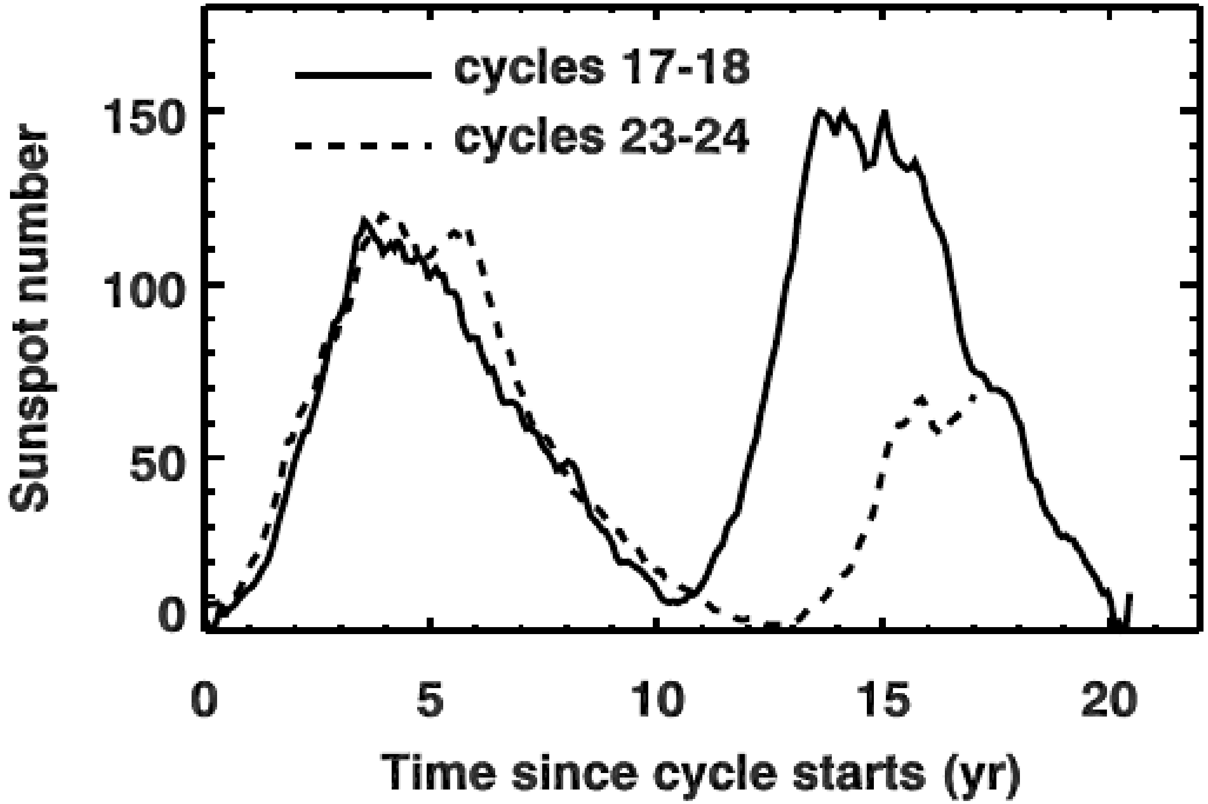
Explained by variations in meridional inflow pattern:

Cameron & Schüssler (2012), Martin-Belda & Cameron (2018)

Surface flux transport (SFT) models with tilt quenching reproduce observed variations in polar field well — except cycle 24

RANDOM FLUCTUATIONS

Effect of scatter in Joy's law considered by Jiang et al. (2014).



Random fluctuations in Joy's law ⇒ unpredictable deviations.

Total poloidal flux $\sim$ surface flux $\Rightarrow$ a single large AR can make a difference

A bipolar AR contributes $$\delta D_{\mathrm{BMR}} \approx F\, d\, \sin\alpha\, \sin\theta$$

$\Rightarrow$ to make a difference, an AR needs to be

- large
- unusually tilted (esp. non-Joy/non-Hale — or very "overJoy")
- close to the equator (?)

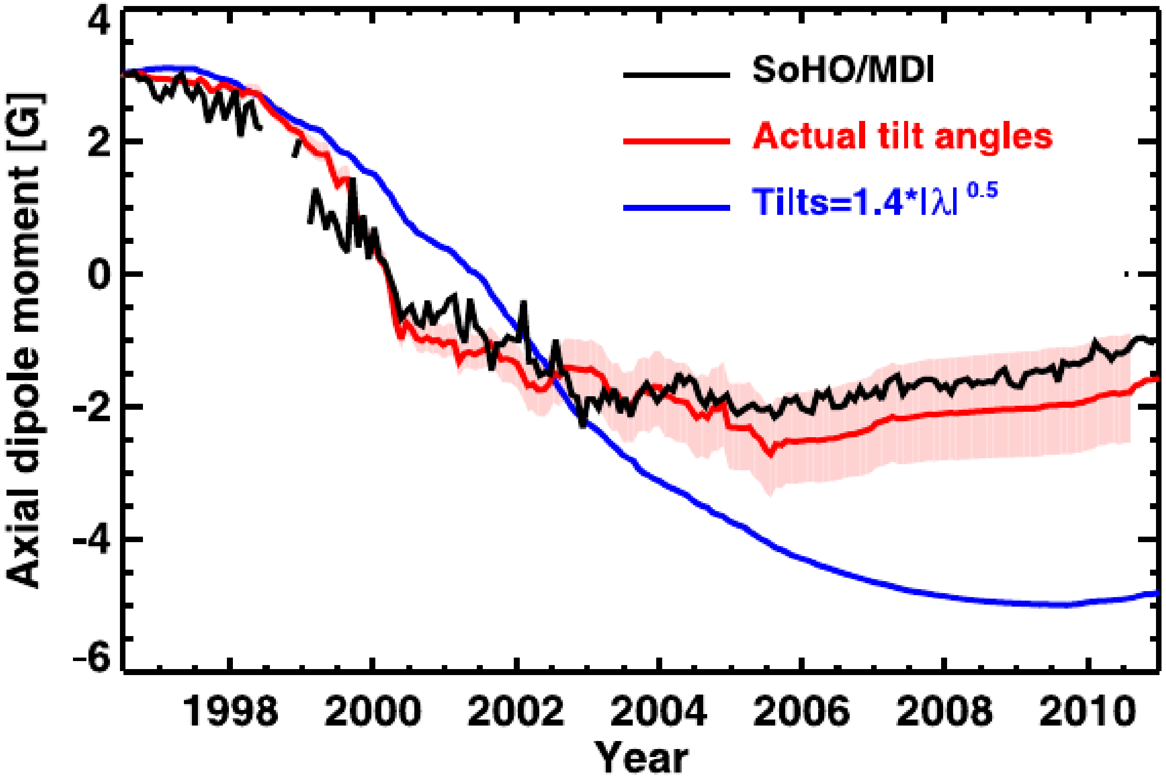
Such "rogue" active regions can play havoc with the cycle. (Cameron+ 2013)

Adjective [edit]

rogue (*comparative* **more rogue**, *superlative* **most rogue**)

1. (*of an animal, especially an elephant*) Vicious and solitary.
2. (*by extension*) Large, destructive and unpredictable.
3. (*by extension*) Deceitful, unprincipled. [quotations ▼]
4. Mischievous, unpredictable. [quotations ▼]

Cycle 23/24 explained as a 2σ fluke due to rogue low-latitude ARs:



Jiang et al. (2015)

Theoretical background: Cameron & Schüssler (2015)

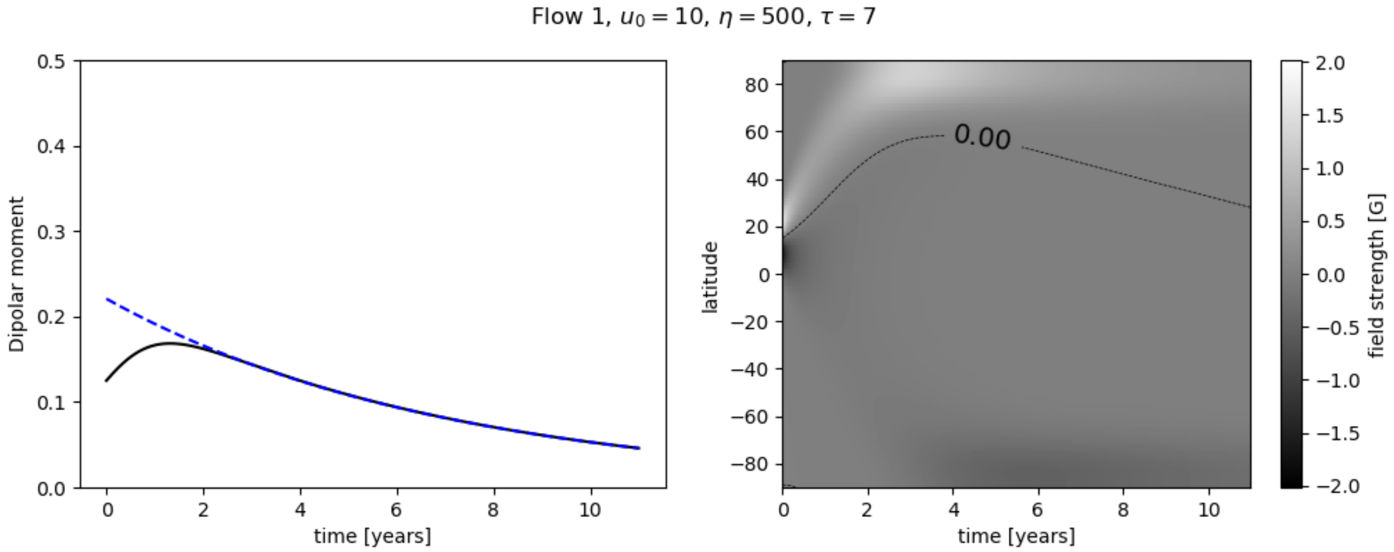
Effect of rogue AR larger at low latitudes (as leading flux can then be cancelled across the equator).

WHAT MAKES A ROGUE AR?

A bipolar AR contributes

$$\delta D_i = \frac{3}{4\pi R^2} \Phi\, d\, \sin\alpha\, \cos\lambda$$

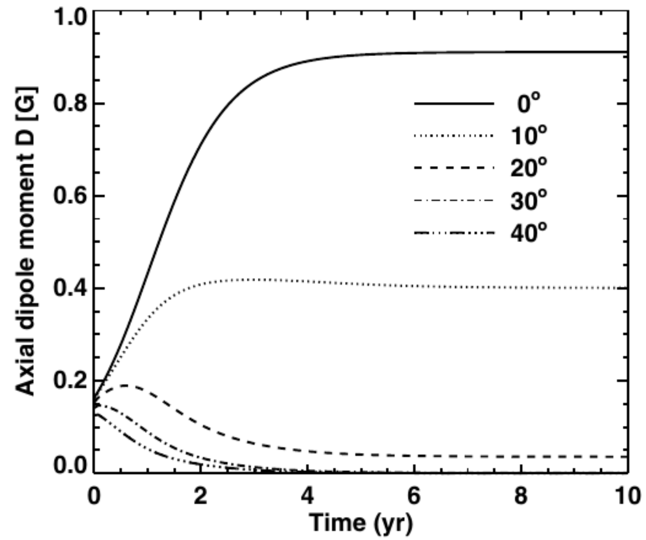
δD_i is only the *initial* dipole contribution.
To evaluate final contribution δD_f, SFT is needed:

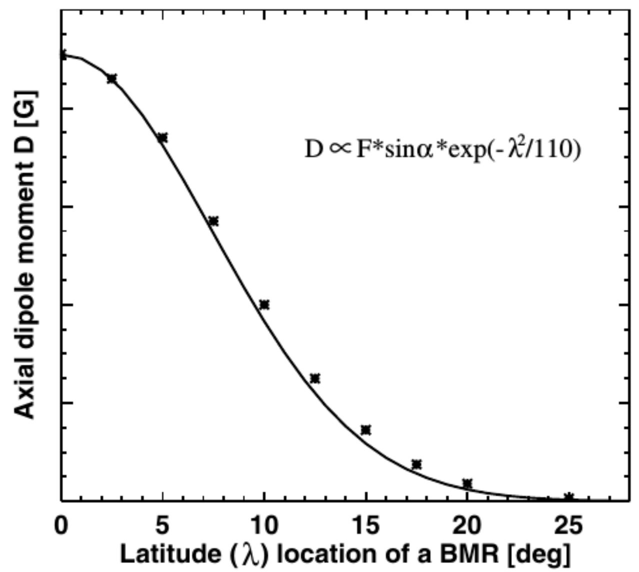


Note that for finite τ, δD will keep decreasing exponentially (dashed)
$\Rightarrow$ here, δD_f will be meant without the factor $e^{-t/\tau}$.

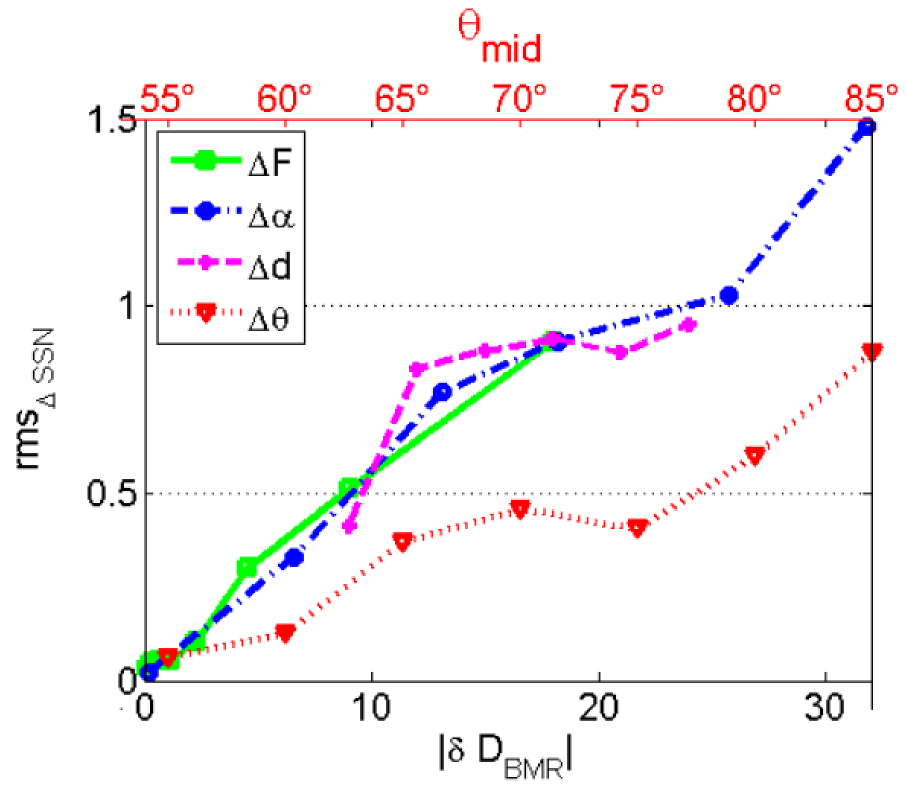
Dependence of δD_f on latitude: conflicting “anecdotal evidence”:

Jiang et al. (2014) SFT:

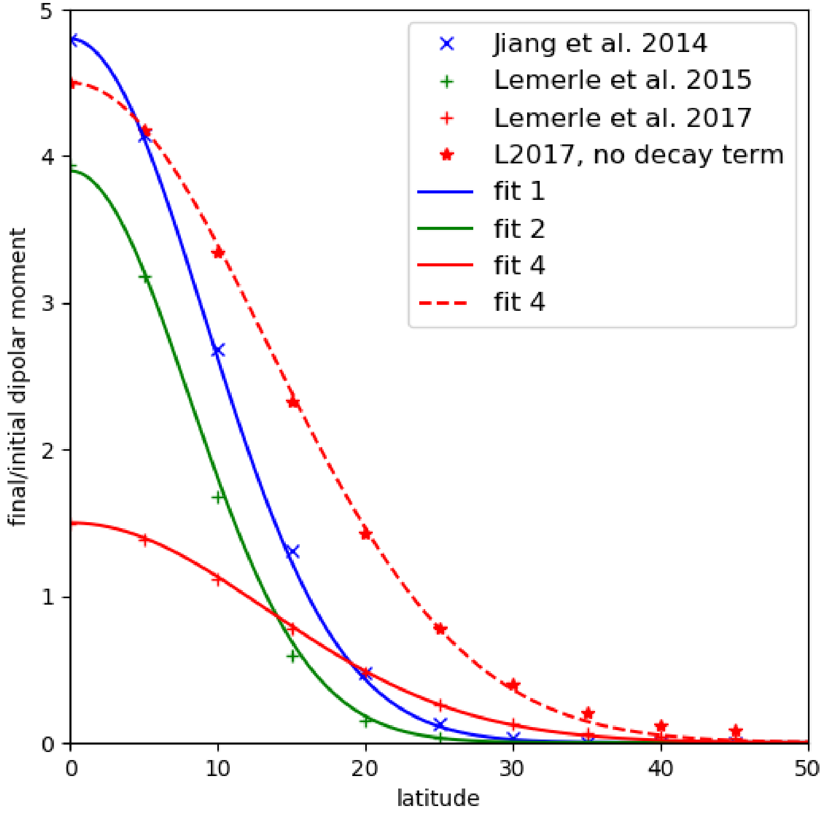




In 2×2D dynamo (Nagy et al. 2017):



Further experiments in a 1D SFT model:



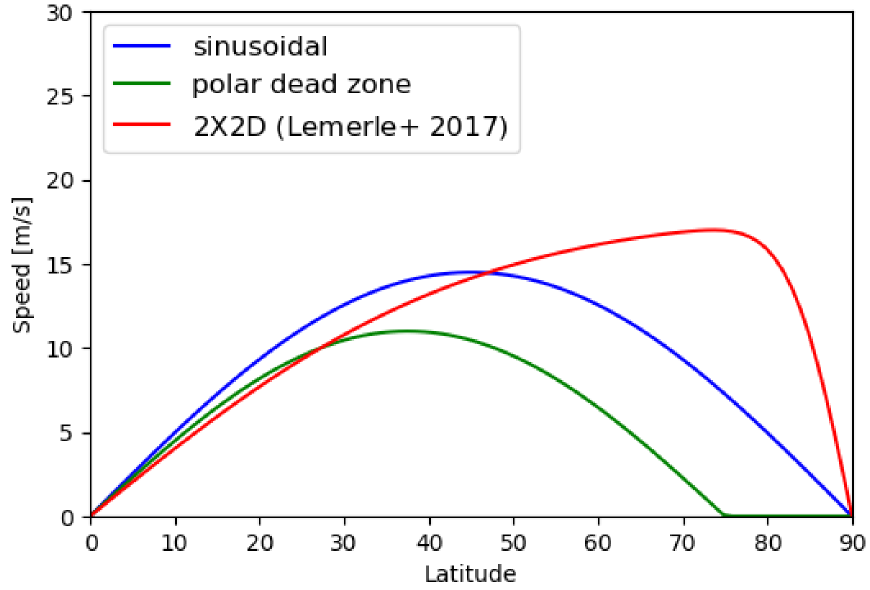
Gaussian dependence
on AR latitude.

Width: dynamo effectivity range λ_D.

A: amplitude.

Let's take a more comprehensive look at this based on an SFT model grid!

$\boxed{\text{Flow profile:}}$ 3 cases considered



Profile 1: sinusoidal

used e.g. by Dikpati et al. (2006):

$$u_c = u_0 \sin(2\lambda)$$

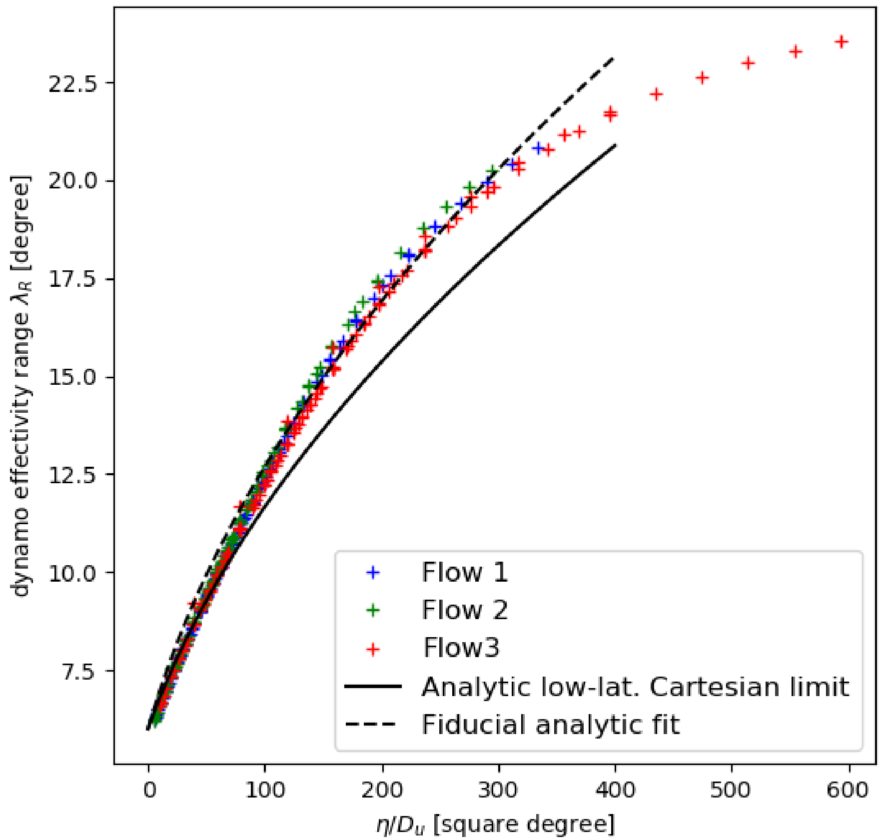
Profile 2: polar dead zone —used e.g. by Jiang et al. (2011–)

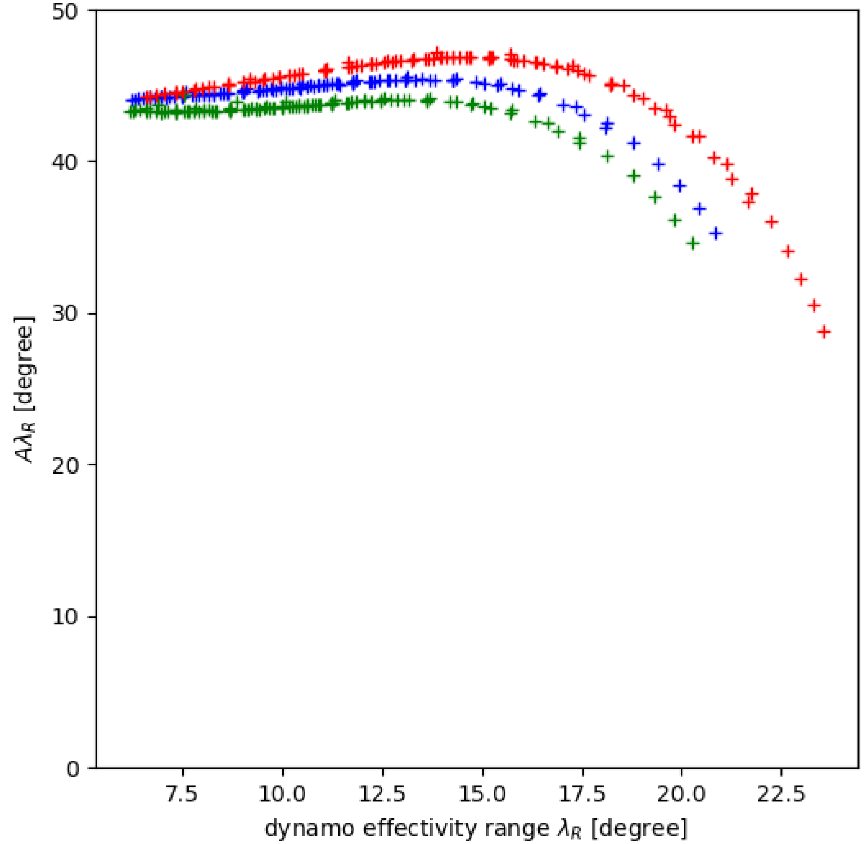
$$u_c = \begin{cases} u_0 = \sin(\pi\lambda/\lambda_0) & \text{if } |\lambda| < \lambda_0 \\ 0 & \text{otherwise} \end{cases}$$

Profile 3: 2×2D —used in Lemerle et al. (2017)

$$u_c(R,\theta) = u_0\,\mathrm{erf}\,(V\cos\lambda)\,\mathrm{erf}\,(\sin\lambda) \qquad V = 7$$

Any system in the madness? Let D_u denote flow divergence on equator:

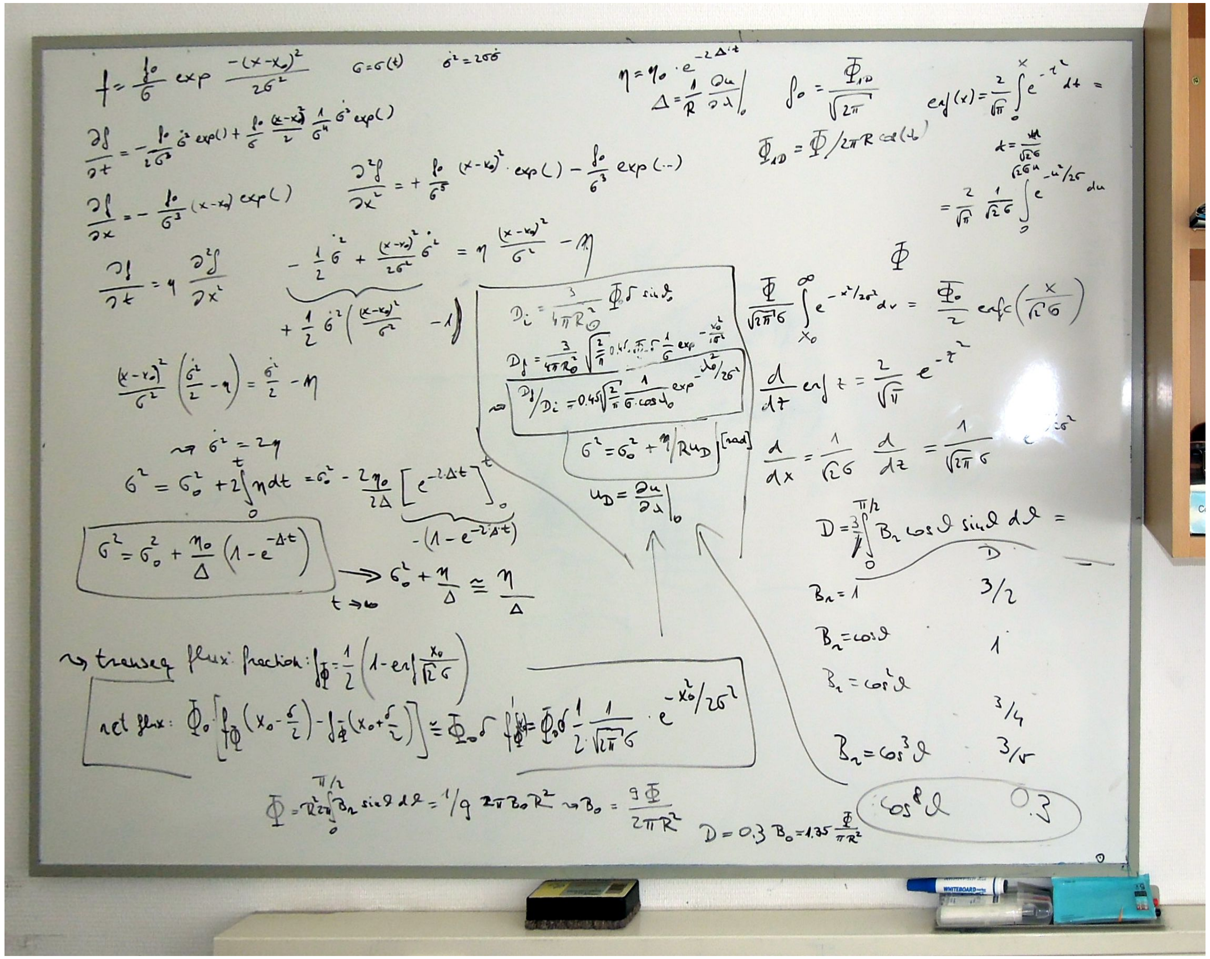




Nice lineup —any deeper reason?

The Gaussian latitudinal cutoff in AR dynamo effectivity:

analytical derivation



Recall initial cond: a pair of flux rings w.Gaussian profile at latitude λ_0, half width $\sigma_0 = 6^\circ$, N–S separation $\delta = 4^\circ$. (Other initial profile will also soon approach Gaussian by virtue of central limit theorem.)

Needed: transequatorial flux in $t \to \infty$ limit

(= flux that will not cancel on the advective time scale).

Consider low latitude limit (λ, $\sigma \ll 1$ radian):

Cartesian geometry, flow divergence $D_u = du/d\lambda \simeq$ const.

Analogy: 1D Hubble flow in a vacuum-dominated universe:

exponential expansion.

In Lagrangian (comoving, expanding) frame λ, $\sigma =$ const. for $\eta = 0$.

For $\eta \neq 0$: $\eta_L = f(t)$ is time dependent in Lagrangian frame: $\eta_L \propto e^{-2D_u t}$.

Self-similar solution of diffusion eq. with this time-dependent η:

$$A\exp-\frac{(\lambda-\lambda_0)^2}{2\sigma^2} \text{ with } \sigma(t)=\left[\sigma_0^2+\frac{\eta}{D_u}(1-e^{-2D_ut})\right]^{1/2} \rightarrow \boxed{\lambda_D=\left(\sigma_0^2+\frac{\eta}{D_u}\right)^{1/2}}$$

The fraction of flux of one polarity across the equator is

$f_\Phi(\lambda_0)=\frac{1}{2}\left(1-\mathrm{erf}\,(\lambda_0/\sqrt{2}\sigma\right)$. The net transequatorial flux is then

$$f_\Phi(\lambda_0-\delta/2)-f_\Phi(\lambda_0+\delta/2)\simeq\frac{\delta}{2^{3/2}\pi^{1/2}\sigma}\exp\frac{-\lambda_0^2}{2\sigma^2}$$ [Taylor exp., leading term]

With $\sigma\rightarrow\lambda_D$, final dipole moment still also depends on flux distribution, but using the observational constraint $B\sim\cos^8\theta$ this free factor can be constrained, finally resulting in a dynamo effectivity factor

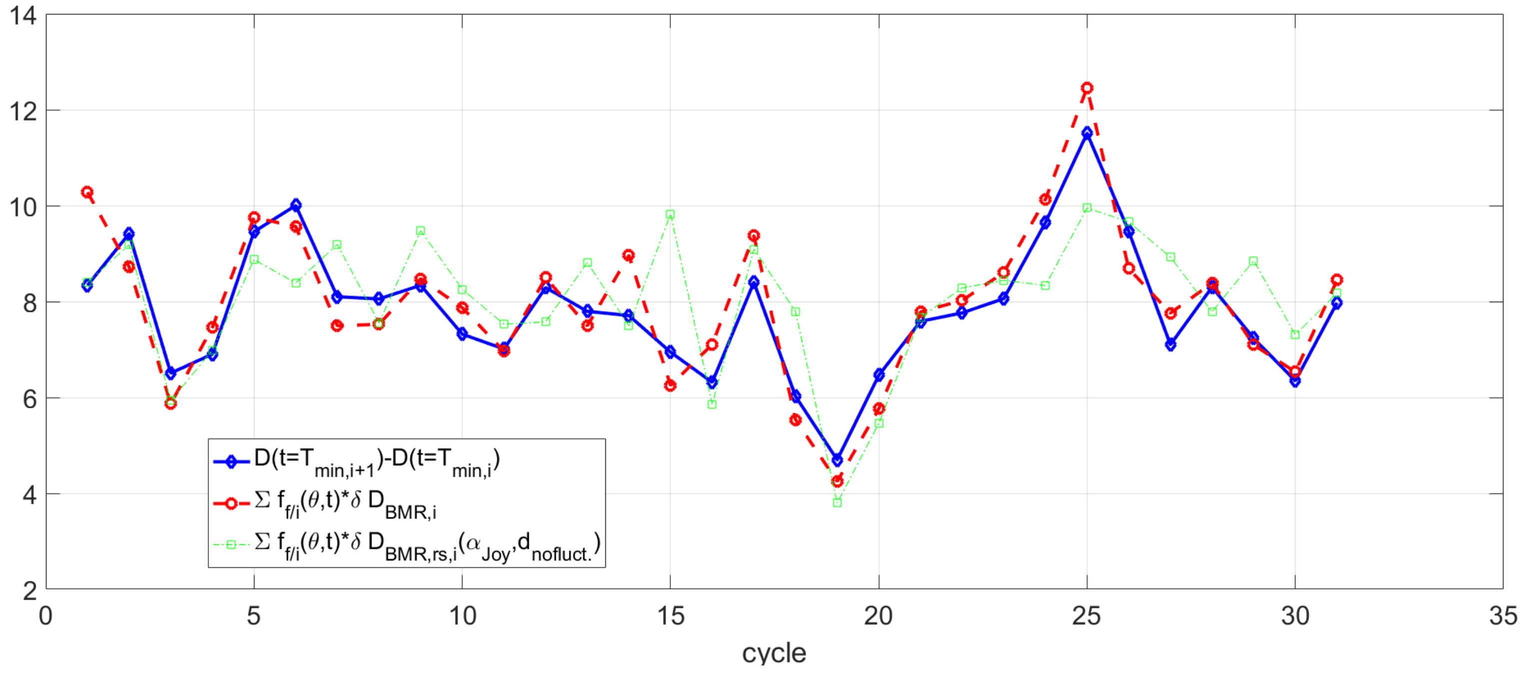
$$\boxed{\delta D_f/\delta D_i=A\exp-\frac{\lambda_0^2}{2\lambda_D^2}/\cos(\lambda_0)} \text{ with } \boxed{A\propto 1/\lambda_D}.$$

⇒ solution of the SFT partial diff.eq. can be bypassed and substituted by an algebraic summation (as done also by Jiang et al. 2019):

$$D_{i+1} - D_i = \sum_{n=1}^{N} f_{fi,n}\,\delta D_n \qquad f_{fi} = (\delta D_f/\delta D_i)\, e^{-(t_{i+1}-t_n)/\tau}$$

Only 3 parameters — no need to worry about the choice of a flow profile!

f_{fi} comes from a 1D SFT model but confirmed in a comparison with the 2D SFT component of the 2×2D dynamo model (Lemerle et al. 2017):



Problem 1: Parameters D_u, η, τ need to be determined.

Petrovay & Talafha (2019): SFT optimization for polar field variation:
pole-reaching flows favor higher η and lower u_0, to reproduce observed $\sin^8\theta$ field profile.

λ_D depends on η/D_u only, increasing monotonically
$\Rightarrow$ pole-reaching flows will result in higher dynamo effectivity.

$\Rightarrow$ effective flow velocity in polar region is important to pin down!
(e.g. Solar Orbiter, EST...)

Problem 2: $N \sim 3000$ in a cycle — still pretty tedious...

~~How many ARs do we need to predict the solar dipole moment?~~

How many ARs do we need to **explain the deviation of** the solar dipole moment **from the value expected for a cycle of given form and amplitude**?

Possible answers:

(a) Zero [Dasi-Espuig 2010, Cameron et al. 2010] —fails for Cyc.24

(b) Hundreds (Whitbread et al. 2018) — overkill as most of those can be substituted by their statistical average.

(c) a low number (Jiang et al. 2015, Nagy et al. 2017)

We approach this problem with ARDoR.

ARDoR IN SOLAR CYCLE PREDICTION

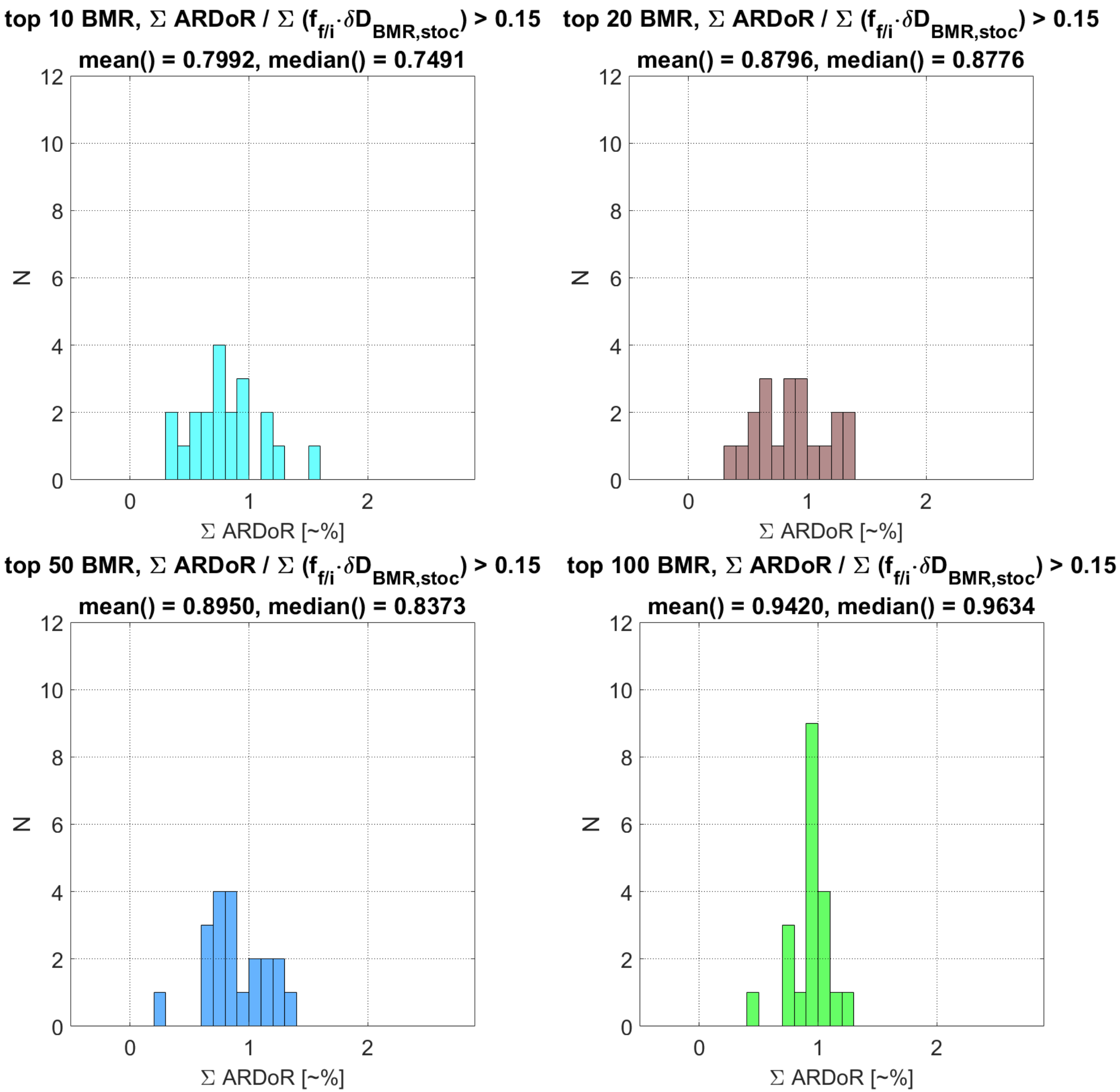
ARDoR = Active Region Degree of Rogueness: $f_{fi}\,(\delta D_i - \delta D_{i,RS})$

RS refers to "reduced stochasticity": an AR of the same size, appearing at the same time and latitude, as observed, but with tilt and separation substituted by their mean values for the given latitude and flux.

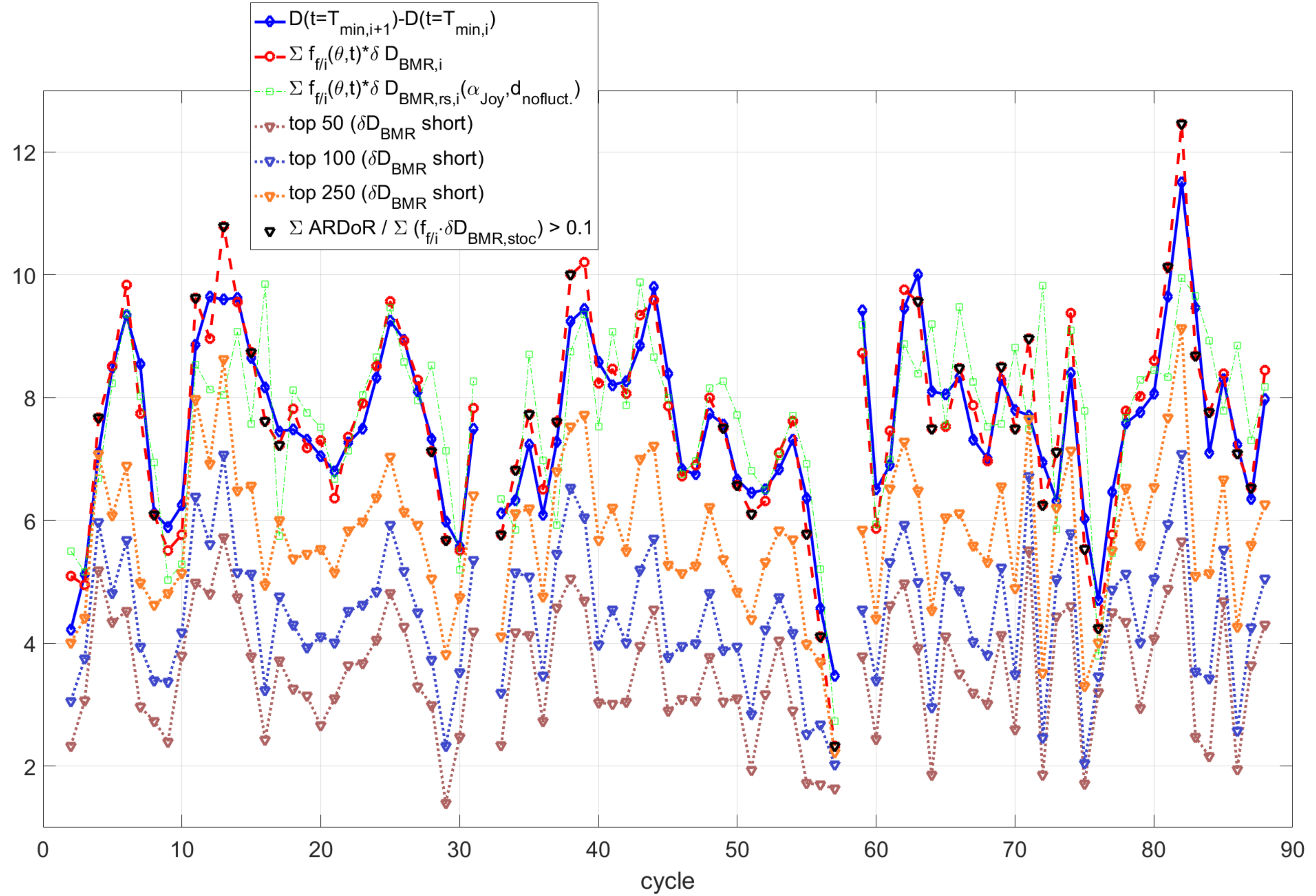
Then, the relative deviation of D at end of cycle from the expected [RS] value is $\Delta = \sum ARDoR/D_{i+1,RS}$.

If Δ is small, no worries. But what if not ?

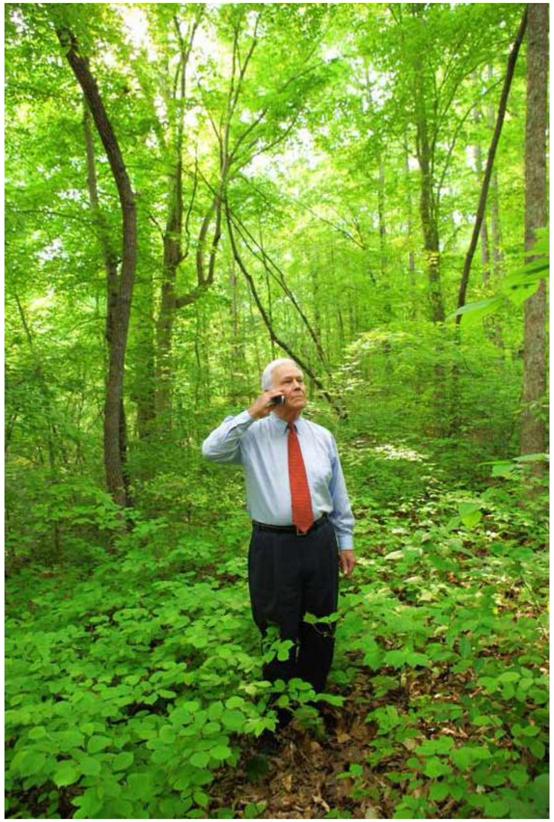
Order ARs by decreasing ARDoR and see (in the 2×2D model)
how many are needed to [roughly] reproduce Δ in those cycles where $\Delta > 0.15$



$\Rightarrow$ 10–20 AR with the highest ARDoR can account for $> 80\,\%$ of the deviation.



D(t=T_min,i+1)-D(t=T_min,i)
Σ f_f/i(θ,t)*δ D_BMR,i
Σ f_f/i(θ,t)*δ D_BMR,rs,i(α_Joy,d_nofluct.)
top 50 (δD_BMR short)
top 100 (δD_BMR short)
top 250 (δD_BMR short)
Σ ARDoR / Σ (f_f/i·δD_BMR,stoc) > 0.1
12
10
8
6
4
2
0
10
20
30
40
50
60
70
80
90
cycle



Lost in the forest! 10–20 is still not “a few”.

Potential way out: reduced stochasticity still has stochasticity in distribution of fluxes, latitudes and emergence times ($\sim$ shot noise).

Next goal: address this.

Furthermore: we need a method that works “on the fly”
—cannot wait for the cycle to finish.